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Large deviation principles for 3D stochastic primitive equations

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Abstract

In this paper, we establish the Freidlin–Wentzell's large deviations for 3D stochastic primitive equations with small noise perturbation. The weak convergence approach plays an important role.

MSC: primary 60F10; secondary 60H15

Keywords: Primitive equation; Large deviations; Weak convergence method

1. Introduction

The large-scale motion of the ocean can be well modeled by 3D viscous primitive equations, which are derived from the Navier–Stokes equations, with rotation, coupled with thermodynamics and salinity diffusion-transport equations, by assuming two important simplifications: Boussinesq approximation and the hydrostatic balance (see [16,17,21]). As a fundamental model in meteorology, this model has been intensively investigated because of the interests stemmed

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from physics and mathematics. For example, the mathematical study of the primitive equations originated in a series of articles by Lions, Temam and Wang in the early 1990s (see [16–19]), where they set up the mathematical framework and showed the global existence of weak solutions. Cao and Titi developed a beautiful approach to dealing with the L^6-norm of the fluctuation $\tilde{v}$ of horizontal velocity and obtained the global well-posedness for the 3D viscous primitive equations in [3].

Along with the great successful developments of these deterministic primitive equations, the random situation has also been developed rapidly. In [15], Guo and Huang obtained the existence of universal random attractor of strong solution for the equations that the momentum equation is driven by an additive stochastic forcing and the thermodynamical equation is driven by a fixed heat source. Debussche, Glatt-Holtz, Temam and Ziane established the global well-posedness of strong solution if the primitive equations are driven by multiplicative stochastic forcing in [7]. In [9], we proved the existence of global weak solutions for 3D stochastic primitive equations driven by regular multiplicative noise, and also obtained the exponential mixing property for weak solutions which are limits of spectral Galerkin approximations.

In this paper, we are concerned with the Freidlin–Wentzell's large deviation principle (LDP) for the stochastic primitive equations, which deals with path probability asymptotic behavior for stochastic dynamical systems with small noise. An important tool for studying the Freidlin–Wentzell's LDP is the weak convergence approach, which is developed by Dupuis and Ellis in [10]. The key idea of this approach is to prove some variational representation formula about the Laplace transform of bounded continuous functionals, which will lead to the proof of equivalence between LDP and Laplace principle. In particular, for Brownian functionals, an elegant variational representation formula has been established by Boué and Dupuis [1], Budhiraja and Dupuis [2].

In the past two decades, there are numerous important results about LDP for stochastic partial differential equations (SPDEs). For example, Cardon-Weber [5] studied Burgers' type SPDEs and achieved their LDP in 1999. In 2004, Cerrai and Röckner established LDP for stochastic reaction–diffusion equations with nonlinear reaction term in [6]. In 2009, under very general conditions, Liu [20] obtained the LDP for SPDEs with monotone coefficients and small multiplicative noise, which covers many models such as stochastic reaction–diffusion equations, stochastic porous media equations and fast diffusion equations, and the stochastic p-Laplace equation in Hilbert space. Gao and Sun [12] proved LDP of weak solution of the two dimensional primitive equations on the state space $C([0, T]; H)$.

The purpose of this paper is to establish LDP of the strong solution of 3D stochastic primitive equations by using the weak convergence method. Note that in [12] the authors considered the weak solution in two dimensional case, they used the state space $C([0, T]; H)$ which is not suitable for our case. To deal with this problem, we change the state space to be $C([0, T]; H^1)$ (see definition below), which leads to some higher Sobolev norm estimates and much more complicated calculation. For instance, during the procedure to prove the global well-posedness of skeleton equation, some non-trivial estimates, such as L^{10}, H^1 estimates are needed. Moreover, it's worth mentioning that our result is obtained without adding any additional regular conditions on the noise, only those in [7] are enough. Our main result is

Theorem 1.1. *Suppose **Hypothesis H0** holds. Then for any $Y_0 \in V$, $\{Y^\varepsilon\}$ satisfies the large deviation principle on $C([0, T]; V) \cap L^2([0, T]; D(A))$ with a good rate function given by (4.19).*

Hypothesis H0 and all the above symbols will be given in the following section.

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