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Dynamics of wave equations with moving boundary

To Fu Ma [a,*], Pedro Marín-Rubio [b], Christian Manuel Surco Chuño [c]

[a] *Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, 13566-590 São Carlos, SP, Brazil*

[b] *Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla, Apdo. de Correos 1160, 41080 Sevilla, Spain*

[c] *Câmpus de Curitiba, Universidade Federal Tecnológica do Paraná, 80230-901 Curitiba, PR, Brazil*

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Abstract

This paper is concerned with long-time dynamics of weakly damped semilinear wave equations defined on domains with moving boundary. Since the boundary is a function of the time variable the problem is intrinsically non-autonomous. Under the hypothesis that the lateral boundary is time-like, the solution operator of the problem generates an evolution process $U(t,\tau): X_\tau \to X_t$, where X_t are time-dependent Sobolev spaces. Then, by assuming the domains are expanding, we establish the existence of minimal pullback attractors with respect to a universe of tempered sets defined by the forcing terms. Our assumptions allow nonlinear perturbations with critical growth and unbounded time-dependent external forces.

MSC: 35R37; 35L70; 35B41

Keywords: Wave equation; Non-cylindrical domain; Non-autonomous system; Pullback attractor; Critical exponent

1. Introduction

This paper is concerned with long-time dynamics of semilinear wave equations defined on moving boundary domains. The problem involves a space–time domain

* Corresponding author.
E-mail addresses: matofu@icmc.usp.br (T.F. Ma), pmr@us.es (P. Marín-Rubio), christianchuno@utfpr.edu.br (C.M. Surco Chuño).

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$$Q_\tau \subset \mathbb{R}^3 \times (\tau, \infty), \quad \tau \in \mathbb{R},$$

such that its intersections with hyperplanes $\{(x,s) \in \mathbb{R}^4 \,|\, s = t\}$ are bounded domains $\Omega_t \subset \mathbb{R}^3$ with boundary $\Gamma_t = \partial\Omega_t$. Then Q_τ and its lateral boundary Σ_τ can be defined as

$$Q_\tau = \bigcup_{t>\tau} \{\Omega_t \times \{t\}\} \quad \text{and} \quad \Sigma_\tau = \bigcup_{t>\tau} \{\Gamma_t \times \{t\}\},$$

respectively. Since Ω_t varies with respect to t we see that Q_τ is, in general, non-cylindrical along the t-axis. We consider the mixed problem

$$\partial_t^2 u - \Delta u + \partial_t u + f(u) = g \text{ in } Q_\tau, \tag{1.1}$$

$$u = 0 \text{ on } \Sigma_\tau, \tag{1.2}$$

$$u(x,\tau) = u_\tau^0(x), \quad \partial_t u(x,t)|_{t=\tau} = u_\tau^1(x), \quad x \in \Omega_\tau, \tag{1.3}$$

where f and $g = g(x,t)$ are forcing terms and u_τ^0 and u_τ^1 are initial data. Sometimes we write simply Q instead of Q_τ.

This kind of wave equation was studied by several authors with $\tau = 0$. Indeed, the existence of a global solution was proved by Cooper and Bardos [10] under the assumption that there exists a one-to-one mapping transforming Q onto an expanding or contracting domain Q^*. One says that a domain Q is expanding if $\Omega_s \subset \Omega_t$ whenever $s \le t$ and contracting in the reverse case. However, uniqueness of solutions is only known under the assumption that the exterior normal to Σ does not belong to the corresponding light cone, as proved in [10]. Writing the exterior normal as $\nu = (\nu_x, \nu_t)$ this implies that $|\nu_t| < |\nu_x|$ on Σ, which defines Σ as time-like. Roughly speaking, under suitable assumptions on f and g, problem (1.1)–(1.3) has a unique global solution if Q is smooth and its lateral boundary Σ is time-like.

On the other hand, the study of long-time dynamics is concerned with the behavior of the solutions as $t \to \infty$. In this direction, it was proved by Bardos and Chen [2] that the linear energy of the system increases when the domain Q is contracting and decreases when the domain is expanding. Therefore if we consider dissipative systems, it is natural to assume that Q is non-contracting. It is not clear whether a damping term can overcome the growth of energy produced by strictly contracting domains. The assumption that Q is expanding is used in the proof of an energy inequality (see Lemma 2.3 below).

Now, since the boundary of Ω_t is a function of time, it follows that evolution equations on moving boundary domains are intrinsically non-autonomous, even if the external force $g(x,t) = g(x)$ does not depend on t. In addition, given initial data (u_τ^0, u_τ^1) in $H_0^1(\Omega_\tau) \times L^2(\Omega_\tau)$, the (finite energy) solutions u of (1.1)–(1.3) satisfy

$$u(t) \in H_0^1(\Omega_t) \text{ and } \partial_t u(t) \in L^2(\Omega_t), \ \forall t \ge \tau,$$

where $u(t)$ denotes $u(\cdot, t)$. Therefore, the solution operator of (1.1)–(1.3) generates an evolution process

$$U(t,\tau) : X_\tau \to X_t, \quad t \ge \tau,$$

where

$$X_t = H_0^1(\Omega_t) \times L^2(\Omega_t).$$