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On the 2-dimensional dual Minkowski problem

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Abstract

We study the 2-dimensional dual Minkowski problem, which is the following nonlinear problem on unit circle

$$u'' + u = g(\theta)u^{-1}(u^2 + u'^2)^{(2-k)/2},\ \theta \in \mathbb{S},$$

for any given positive continuous function $g(\theta)$ with $2\pi/m$-periodic. We prove that it is solvable for all $k \in (1, +\infty)$ and $m \in \{3, 4, 5 \cdots\}$.

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1. Introduction

A Minkowski problem is to establish the necessary and sufficient conditions for a given finite Borel measure to arise as a measure generated by a convex body, which includes, for example, the surface area measure of a convex body in the classical Minkowski problem [2,29], the L_p surface area measure of a convex body in the L_p Minkowski problem [27], and the dual curvature

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measure in the dual Minkowski problem introduced recently by Huang etc. [18]. Analytically, the studying of a Minkowski problem is equivalent to studying a degenerate Monge–Ampère equation [30–32]. Let $\mathbb{S}^{n-1}$ be the unit sphere in $\mathbb{R}^n$, and $u:\mathbb{S}^{n-1}\to(0,+\infty)$ be the unknown supporting function of a convex body. Denote by $e_{i,j}$ the standard Riemannian metric on $\mathbb{S}^{n-1}$, and g the density function of a finite Borel measure on $\mathbb{S}^{n-1}$, then the classical Minkowski problem is equivalent to the Monge–Ampère equation

$$det(u_{ij}+e_{i,j}u)=g(v),\ v\in\mathbb{S}^{n-1},$$

which has led to a series of important works [5,6,8,30,31]. The L_p Minkowski problem given by Lutwak [27] is a natural L_p extension of the classical Minkowski problem. For a fixed $p\in\mathbb{R}$, if the given Borel measure on $\mathbb{S}^{n-1}$ has a density function g. Then the L_p Minkowski problem can be formulated by a fully nonlinear partial differential equation on sphere $\mathbb{S}^{n-1}$, namely

$$u^{1-p}det(u_{ij}+e_{ij}u)=g(v), v\in\mathbb{S}^{n-1}. \tag{1.1}$$

This problem has attracted extensive attention, and many important results have been obtained, see [1,4,7,9,11,16,15,17,20,21,23,22,24,26–28,37–39] and their references.

Recently, Huang et al. [18] introduced the dual curvature measure and studied the existence for the dual Minkowski problem under the assumptions that $1\le k\le n$ for the k-th dual curvature measure and that the given Borel measure is even. Moreover, if the given Borel measure has a density function $g:\mathbb{S}^{n-1}\to\mathbb{R}$, then the dual Minkowski problem relates to the following fully nonlinear elliptic equation

$$u(u^2+|\nabla u|^2)^{\frac{k-n}{2}}det(u_{ij}+e_{ij}u)=g(v),\ v\in\mathbb{S}^{n-1}, \tag{1.2}$$

where $k\in\mathbb{R}$ and ∇u denotes the gradient vector of u respect to a frame on $\mathbb{S}^{n-1}$. It is clear that equation (1.2) with $k=n$ is the same as (1.1) with $p=0$, which is related to the logarithmic Minkowski problem [4,33,37]. Li et al. [25] obtained the regularities of the dual Minkowski problem by a flow method. From [18] (see also [25]), equation (1.2) may be seen as the Euler–Lagrange equation with respect to the functional

$$\mathbb{F}(K)=\begin{cases}\int_{\mathbb{S}^{n-1}}g(v)\log u_K dv-\int_{\mathbb{S}^{n-1}}\log r_K(\xi)d\xi,\\ \int_{\mathbb{S}^{n-1}}g(v)\log u_K dv-\frac{1}{k}\int_{\mathbb{S}^{n-1}}r_K^k(\xi)d\xi,\ k\neq 0,\end{cases}$$

where (u_K, r_K) is the support function and radial function of convex body K.

In this paper, we study the existence of solutions to the 2-dimensional dual Minkowski problem for all $k>1$ via studying the existence of positive solutions to (1.2) with $n=2$. We deal with a more general case, by introducing a parameter $l\in[0,1]$ as the coefficient of the term $|\nabla u|^2$ in equation (1.2) and then derive the following nonlinear problem

$$u''(\theta)+u(\theta)=g(\theta)u^{-1}(u^2+lu'^2)^{(2-k)/2},\ \theta\in\mathbb{S}, \tag{1.3}$$

where $g(\theta)$ is a continuous $2\pi/m$-periodic function with $m\in\mathbb{N}$, $k\in\mathbb{R}$, and $l\in[0,1]$ is a parameter. The special form of (1.3) with $l=0$ is equivalent to the 2-dimensional L_p Minkowski problem with $p=2-k$; While (1.3) with $l=1$ relates to the 2-dimensional dual Minkowski

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