



A necessary and sufficient condition for well-posedness of initial value problems of retarded functional differential equations

Junya Nishiguchi

Department of Mathematics, Kyoto University, Kitashirakawa Oiwake-cho, Sakyo-ku, Kyoto 606-8502, Japan

Received 5 December 2016; revised 26 April 2017

Abstract

We introduce the retarded functional differential equations (RFDEs) with general delay structure to treat various delay differential equations (DDEs) in a unified way and to clarify the delay structure in those dynamics. We are interested in the question as to which space of histories is suitable for the dynamics of each DDE, and investigate the well-posedness of the initial value problems (IVPs) of the RFDEs. A main theorem is that the IVP is well-posed for any “admissible” history functional if and only if the semigroup determined by the trivial RFDE $\dot{x} = 0$ is continuous. We clarify the meaning of the Hale–Kato axiom (Hale & Kato [12]) by applying this result to RFDEs with infinite delay. We also apply the result to DDEs with unbounded time- and state-dependent delays.

© 2017 Elsevier Inc. All rights reserved.

Keywords: Retarded functional differential equations; Well-posedness of initial value problems; Infinite delay; State-dependent delay

Contents

1. Introduction	2
2. Introduction to general delay structure	5
2.1. Continuations and continuability	6
2.1.1. Continuations	6

E-mail address: j-nishi@math.kyoto-u.ac.jp.

<http://dx.doi.org/10.1016/j.jde.2017.04.038>

0022-0396/© 2017 Elsevier Inc. All rights reserved.

Please cite this article in press as: J. Nishiguchi, A necessary and sufficient condition for well-posedness of initial value problems of retarded functional differential equations, J. Differential Equations (2017), <http://dx.doi.org/10.1016/j.jde.2017.04.038>

2	<i>J. Nishiguchi / J. Differential Equations</i> ●●● (●●●●) ●●●—●●●	
	2.1.2. Continuability	7
	2.1.3. Examples	9
	2.2. Trivial RFDE	9
	2.3. Comments on preceding results about continuability	10
	2.3.1. Axiom by Hale & Kato	10
	2.3.2. Hypotheses by Schumacher	12
3.	Uniqueness and maximal solutions	14
	3.1. Maximality	14
	3.2. Solution process	16
4.	Existence and uniqueness theorems	16
	4.1. A variation of the Picard–Lindelöf theorem for RFDEs	18
	4.1.1. Lipschitzian about continuations	18
	4.1.2. Lipschitzian about memories	20
	4.2. Cauchy–Peano existence theorem for RFDEs	21
5.	A necessary and sufficient condition for well-posedness	23
	5.1. Well-posedness of the initial value problem	23
	5.2. Examples	27
6.	Applications	28
	6.1. Proof of well-posedness result under the Hale–Kato axiom	29
	6.2. Well-posedness for differential equations with time- and state-dependent delays	30
	Acknowledgments	34
	Appendix A. Local semiflows on topological spaces	34
	A.1. Local semiflows and local processes	34
	A.2. Continuity of local semiflows and local processes	36
	Appendix B. Semi-groups of linear operators	41
	References	41

1. Introduction

A differential equation of a dependent variable $x \in \mathbb{E}$ with one independent variable $t \in \mathbb{R}$ is called a *delay differential equation* (abbrev DDE), if the derivative of an unknown function $x = x(\cdot)$ at t depends on the past $\{x(s) : s < t\}$ before t . Here $\mathbb{E} = (\mathbb{E}, \|\cdot\|)$ is a Banach space. DDEs are classified by types of delay; finite delay and infinite delay. A DDE is said to be of *finite delay* if there is $r > 0$ such that for all t , the derivative $\dot{x}(t)$ only depends on

$$\{x(s) : t - r \leq s \leq t\}.$$

Otherwise, the equation is said to be of *infinite delay*.

A DDE with dependent variable $x \in \mathbb{R}$ is a scalar DDE, and a system of scalar DDEs is an example of a DDE with dependent variable $x \in \mathbb{R}^n$ for some $n \geq 2$. Sometimes, the dependent variables may belong to an infinite dimensional space. A DDE of a dependent variable $x = (x^i)_{i \in \mathbb{Z}}$ on a Banach space $\ell^\infty(\mathbb{Z}, \mathbb{R})$ appears as *lattice differential equation* (LDE) with delay (see Caraballo et al. [2]).

We consider examples of the types of time-delay in DDEs. A differential equation with *constant delay*

$$\dot{x}(t) = f(t, x(t), x(t - r)), \quad r > 0 \tag{1}$$

Download English Version:

<https://daneshyari.com/en/article/5774058>

Download Persian Version:

<https://daneshyari.com/article/5774058>

[Daneshyari.com](https://daneshyari.com)