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A necessary and sufficient condition for well-posedness of initial value problems of retarded functional differential equations

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Abstract

We introduce the retarded functional differential equations (RFDEs) with general delay structure to treat various delay differential equations (DDEs) in a unified way and to clarify the delay structure in those dynamics. We are interested in the question as to which space of histories is suitable for the dynamics of each DDE, and investigate the well-posedness of the initial value problems (IVPs) of the RFDEs. A main theorem is that the IVP is well-posed for any "admissible" history functional if and only if the semigroup determined by the trivial RFDE $\dot{x} = 0$ is continuous. We clarify the meaning of the Hale–Kato axiom (Hale & Kato [12]) by applying this result to RFDEs with infinite delay. We also apply the result to DDEs with unbounded time- and state-dependent delays.

Keywords: Retarded functional differential equations; Well-posedness of initial value problems; Infinite delay; State-dependent delay

Contents

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1. Introduction

A differential equation of a dependent variable $x \in \mathbb{E}$ with one independent variable $t \in \mathbb{R}$ is called a *delay differential equation* (abbrev DDE), if the derivative of an unknown function $x = x(\cdot)$ at t depends on the past $\{x(s) : s < t\}$ before t. Here $\mathbb{E} = (\mathbb{E}, \|\cdot\|)$ is a Banach space. DDEs are classified by types of delay; finite delay and infinite delay. A DDE is said to be of *finite delay* if there is $r > 0$ such that for all t, the derivative $\dot{x}(t)$ only depends on

$$\{x(s) : t - r \leq s \leq t\}.$$

Otherwise, the equation is said to be of *infinite delay*.

A DDE with dependent variable $x \in \mathbb{R}$ is a scalar DDE, and a system of scalar DDEs is an example of a DDE with dependent variable $x \in \mathbb{R}^n$ for some $n \geq 2$. Sometimes, the dependent variables may belong to an infinite dimensional space. A DDE of a dependent variable $x = (x^i)_{i \in \mathbb{Z}}$ on a Banach space $\ell^\infty(\mathbb{Z}, \mathbb{R})$ appears as *lattice differential equation* (LDE) with delay (see Caraballo et al. [2]).

We consider examples of the types of time-delay in DDEs. A differential equation with *constant delay*

$$\dot{x}(t) = f(t, x(t), x(t - r)), \quad r > 0 \tag{1}$$

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