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On a class of quasilinear Schrödinger equations with superlinear or asymptotically linear terms ☆

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Abstract

We study the existence and nonexistence of nonzero solutions for the following class of quasilinear Schrödinger equations:

$$-\Delta u + V(x)u + \frac{\kappa}{2}[\Delta(u^2)]u = h(u), \quad x \in \mathbb{R}^N,$$

where $\kappa > 0$ is a parameter, $V(x)$ is a continuous potential which is large at infinity and the nonlinearity h can be asymptotically linear or superlinear at infinity. In order to prove our existence result we have applied minimax techniques together with careful L^∞-estimates. Moreover, we prove a Pohozaev identity which justifies that $2^* = 2N/(N-2)$ is the critical exponent for this class of problems and it is also used to show nonexistence results.

MSC: 35J20; 35J60; 35J62

Keywords: Quasilinear Schrödinger equation; Superlinear or asymptotically linear nonlinearities; Pohozaev identity; Moser iteration

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1. Introduction and main results

In this paper, we are concerned with the existence and nonexistence of nonzero solution for *quasilinear Schrödinger equations* of type

$$-\Delta u + V(x)u + \frac{\kappa}{2}[\Delta(u^2)]u = h(u) \quad \text{in} \quad \mathbb{R}^N, \tag{1.1}$$

where $N \geq 3$, $V : \mathbb{R}^N \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ are continuous functions and κ is a real parameter. Solutions of (1.1) are related to existence of standing wave solutions for the following modified version of the nonlinear Schrödinger equation:

$$i\frac{\partial \psi}{\partial t} = -\Delta\psi + W(x)\psi - \eta(|\psi|^2)\psi + \frac{\kappa}{2}\left[\Delta\rho(|\psi|^2)\right]\rho'(|\psi|^2)\psi, \tag{1.2}$$

where $\psi : \mathbb{R}^N \times \mathbb{R} \to \mathbb{C}$, κ is a real constant, $W : \mathbb{R}^N \to \mathbb{R}$ is a given potential and $\eta : \mathbb{R}_+ \to \mathbb{R}$, $\rho : \mathbb{R}_+ \to \mathbb{R}$ are appropriate functions. Quasilinear equations of the form (1.2) appear naturally in mathematical physics and have been derived as models of several physical phenomena corresponding to various types of nonlinear terms ρ, see [27]. Here, we are interested in the case $\rho(s) = s$, which was used, for instance, for the superfluid film equation in plasma physics by Kurihara in [22]. When $\eta(s)$ is a pure power, (1.2) also appears in nonlinear optics, e.g., oscillating soliton instabilities during microwave and laser heating of plasma, see [20,28]. For more details and physical applications involving this subject, we refer the readers to [9,12,21,23,24,27] and references therein. Our main interest is in the existence of *standing wave solutions*, that is, solutions of type

$$\psi(x,t) = \exp(-i\mathcal{E}t)u(x),$$

where $\mathcal{E} \in \mathbb{R}$ and $u \geq 0$ is a real function. A simple computation shows that ψ satisfies (1.2) if and only if the function $u(x)$ solves the quasilinear equation (1.1), where $V(x) := W(x) - \mathcal{E}$ is the new potential and $h(u) := \eta(u^2)u$ is the new nonlinear term.

Motivated by these physical aspects, equation (1.1) has attracted a lot of attention of many researchers and some existence and multiplicity results have been obtained. The semilinear case $\kappa = 0$ has been studied extensively in recent years with a huge variety of hypotheses on the potential $V(x)$ and the nonlinearity $h(t)$, see for example [5,7,8,10,29,33] and references therein. Compared to the semilinear case, the quasilinear one ($\kappa \neq 0$) becomes much more complicated due to the effects of the quasilinear and non-convex term $\Delta(u^2)u$. One of the main difficulties of (1.1) is that there is no suitable space on which the energy functional is well defined and belongs to C^1-class, except for $N = 1$ (see [27]). Another feature of the quasilinear equation (1.1), when $\kappa < 0$ and $h(u) = |u|^{p-2}u$, is that the critical exponent is not $2^* = 2N/(N-2)$, the usual Sobolev exponent. Instead, $p = 2.2^*$ behaves like a critical exponent for (1.1) as observed in the works [14,25] and proved in [2] for $V(x)$ constant. To the best of our knowledge, the first existence result involving variational methods was due to [27] for $N = 1$ or V radially symmetric for high dimensions, by using a constrained minimization argument (see also [24] for the more general case). After that, some ideas and approaches were developed to overcome the difficulties, see [24,30] for a Nehari manifold argument. By using a change of variables (dual approach), the authors in [26] reduced the quasilinear equation (1.1) to a semilinear one, and an Orlicz space framework was used to prove the existence of a positive solution via minimax methods. The same

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