ARTICLE IN PRESS YJDEQ:8546

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Global existence and asymptotic behavior for the 3D compressible Navier–Stokes equations without heat conductivity in a bounded domain ☆

Guochun Wu

School of Mathematical Sciences, Huaqiao University, Quanzhou 362021, China

Received 19 April 2016; revised 21 August 2016

Abstract

In this paper, we investigate the global existence and uniqueness of strong solutions to the initial boundary value problem for the 3D compressible Navier–Stokes equations without heat conductivity in a bounded domain with slip boundary. The global existence and uniqueness of strong solutions are obtained when the initial data is near its equilibrium in $H^2(\Omega)$. Furthermore, the exponential convergence rates of the pressure and velocity are also proved by delicate energy methods.

MSC: 76W05; 35Q35; 35D05

Keywords: Navier–Stokes equations; Global existence; Asymptotic behavior; Bounded domain

1. Introduction

Let $\Omega \subset \mathbb{R}^3$ be a smooth bounded domain, we consider the following well-known compressible Navier–Stokes equations for the motion of compressible viscous fluids:

☆ Supported by China Postdoctoral Science Foundation (Grant No. 2015M581187) and the Scientific Research Funds of Huaqiao University.

E-mail address: guochunwu@126.com.

http://dx.doi.org/10.1016/j.jde.2016.09.046
0022-0396/

Please cite this article in press as: G. Wu, Global existence and asymptotic behavior for the 3D compressible Navier–Stokes equations without heat conductivity in a bounded domain, J. Differential Equations (2016), http://dx.doi.org/10.1016/j.jde.2016.09.046

$$\begin{cases} \rho_t + \mathrm{div}(\rho u) = 0, \\ (\rho u)_t + \mathrm{div}(\rho u \otimes u) + \nabla p = \mathrm{div} T, \\ (\rho E)_t + \mathrm{div}(\rho E u + p u) = \mathrm{div}(u T) + \kappa \Delta \theta, \end{cases} \tag{1.1}$$

for $(x,t) \in \Omega \times \mathbb{R}^+$. Here ρ, u, p, θ denote the density, velocity, pressure and temperature respectively. The specific total energy $E = \frac{1}{2}|u|^2 + \mathcal{E}$, $\mathcal{E}$ is the specific internal energy. The stress tensor is given by

$$T = \mu(\nabla u + \nabla u^T) + \lambda(\mathrm{div} u) I.$$

μ and λ are the coefficient of viscosity and second coefficient of viscosity, respectively. κ is the coefficient of heat conduction. In this paper, it will be always assumed that

$$\mu > 0, \quad 3\lambda + 2\mu > 0, \quad \kappa = 0. \tag{1.2}$$

We will consider only polytropic fluids, so that the equations of state for the fluid is given by

$$p = R\rho\theta, \quad \mathcal{E} = c_v \theta, \quad p = A e^{\frac{s}{c_v}} \rho^\gamma, \tag{1.3}$$

where $A > 0$ is a constant, $\gamma > 1$ is the adiabatic exponent, s is the entropy, and $c_v = R/(\gamma - 1)$.

To begin with, we note the fact that all thermodynamics variables $\rho, \theta, \mathcal{E}, p$ as well as the entropy s can be represented by functions of any two of them. To overcome the difficulties arising from the non-dissipation on θ, we will rewrite system (1.1). We take the two variables to be p and s. In light of the state equation (1.3), we deduce that

$$\rho = A^{-\frac{c_v}{c_v+R}} p^{\frac{c_v}{c_v+R}} e^{-\frac{s}{c_v+R}}. \tag{1.4}$$

Under the aforementioned assumptions, we can rewrite the system (1.1) in terms of (p, u, s) as follows:

$$\begin{cases} p_t + \gamma p \mathrm{div} u + u \cdot \nabla p = \frac{\Phi[u]}{c_v}, \\ \rho u_t + \rho u \cdot \nabla u + \nabla p = \mu \Delta u + (\mu + \lambda) \nabla \mathrm{div} u, \\ s_t + u \cdot \nabla s = \frac{\Phi[u]}{p}, \end{cases} \tag{1.5}$$

where $\Phi[u]$ is the classical dissipation function:

$$\Phi[u] = \frac{\mu}{2} |\nabla u + \nabla u^T|^2 + \lambda (\mathrm{div} u)^2. \tag{1.6}$$

It should be mentioned that system (1.5) is a hyperbolic–parabolic system, while the dissipation property comes from viscosity. In this paper, we consider the initial boundary value problem for system (1.5), which is supplemented by the following initial and boundary conditions:

$$\begin{cases} (p, u, s)(x, 0) = (p_0, u_0, s_0), & x = (x_1, x_2, x_3) \in \Omega, \\ u|_{\partial\Omega} = 0, & t \geq 0, \\ \int_\Omega p_0^{\frac{1}{\gamma}} dx \big/ |\Omega| = \bar{p}_0^{\frac{1}{\gamma}} > 0. \end{cases} \tag{1.7}$$