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Regular and slow-fast codimension 4 saddle-node bifurcations

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Abstract

Using geometric singular perturbation theory, including the family blow-up as one of the main techniques, we prove that the cyclicity, i.e. maximum number of limit cycles, in both regular and slow-fast unfoldings of nilpotent saddle-node singularity of codimension 4 is 2. The blow-up technique enables us to use the well known results for slow-fast codimension 1 and 2 Hopf bifurcations, slow-fast Bogdanov–Takens bifurcations and slow-fast codimension 3 saddle and elliptic bifurcations.

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1. Introduction

In planar slow-fast systems $X_{\epsilon,\mu}$ a curve of singularities, called the critical curve, appears for $\epsilon = 0$ where ϵ is a singular perturbation parameter and $\mu \in \mathbb{R}^p$, $\mu \sim 0$. The critical curve typically consists of normally hyperbolic singularities (the linearized vector field at a normally hyperbolic singularity has one zero eigenvalue with corresponding eigenvector tangent to the critical curve) and one contact point (often called turning point). We assume the contact point is of nilpotent type, for $\mu = 0$. It is shown in [12] that any smooth family of planar slow-fast vector fields $X_{\epsilon,\mu}$, locally near the nilpotent contact point for $(\epsilon, \mu) \sim (0, 0)$, is smoothly equivalent (preserving (ϵ, μ)) to the following normal form:

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$$\begin{cases} \dot{x} = y - f(x,\mu) \\ \dot{y} = \epsilon\Big(g(x,\epsilon,\mu) + \big(y - f(x,\mu)\big)h(x,y,\epsilon,\mu)\Big) \end{cases} \tag{1}$$

for smooth functions f, g and h and $f(0,0) = \partial_x f(0,0) = 0$.

Remark 1. In this paper we focus on smooth families of vector fields (smooth stands for C^∞-smoothness).

In this paper, we assume the nilpotent contact point is of order two ($\frac{\partial^2 f}{\partial x^2}(0,0) \neq 0$). After a smooth coordinate change and a smooth rescaling of time (see [11]), the family (1) can be brought into the form

$$\begin{cases} \dot{x} = y \\ \dot{y} = -xy + \epsilon\widetilde{g}(x,\epsilon,\mu) + \epsilon y^2 H(x,y,\epsilon,\mu) \end{cases} \tag{2}$$

where $\widetilde{g}$ and H are smooth functions.

We call the order of vanishing of $\widetilde{g}(x,0,0)$ at $x = 0$, which is ≥ 0, *the singularity order at the contact point* $(x,y) = (0,0)$ (see [12]). The determination of small-amplitude limit cycles (i.e. limit cycles in a fixed neighborhood of the origin $(x,y) = (0,0)$) in planar slow-fast systems (2) has recently been the subject of many investigations, and *the main goal of this paper is to give a complete analysis of the small-amplitude limit cycles in (2) when the singularity order at the contact point is 4*. When the contact point is a slow-fast jump point (i.e. the singularity order is 0), then it is easy to see that there are no limit cycles (see [15,26,32]). If the singularity order is 1, small-amplitude limit cycles may be generated by a (slow-fast) Hopf bifurcation as $\widetilde{g}(0,\epsilon,\mu)$ varies through the origin. Small-amplitude limit cycles in a codimension 1 slow-fast Hopf case have been studied in [26] generalizing the Van der Pol system (see [15]). In [16], a slow-fast Hopf point of higher codimensions in Liénard setting ($H \equiv 0$ in (2)) has been dealt with. The main result in [16] gives finite upper bounds for the number of small-amplitude limit cycles in analytic families or in smooth families with finite codimension. In a general ("non-Liénard") setting, a codimension 2 slow-fast Hopf point, in the presence of center, has been treated in [25]. The maximum number of small-amplitude limit cycles in this case is shown to be 2 (we refer to this paper for more details). When the singularity order at the contact point in (2) is 2, we deal with a slow-fast unfolding of a Bogdanov–Takens point, and it is shown that from this point, at most one limit cycle may perturb (see [10]). This case was easier to treat due to the presence of the symmetry-breaking quadratic term αx^2 ($\alpha \neq 0$) in $\widetilde{g}$. When the singularity order at the contact point is 3, the family (2) is called the slow-fast unfolding of a saddle singularity of codimension 3 (+) or the slow-fast unfolding of an elliptic singularity of codimension 3 (–), depending on the sign in front of the cubic term in $\widetilde{g}$ (see [23]). In analogy with the results for the slow-fast Hopf point, the number of small-amplitude limit cycles in this codimension 3 case depends on the higher order terms in $\widetilde{g}$, and, in the presence of the quartic term αx^4 ($\alpha \neq 0$) in $\widetilde{g}$, it is shown that the maximum number of limit cycles of both the slow-fast saddle point of codimension 3 and the slow-fast elliptic point of codimension 3 is 2. This cyclicity result follows from [23–25]. The cases with the singularity order at the contact point ≥ 4 have not yet been studied and, as mentioned above, in this paper we investigate the small-amplitude limit cycle phenomenon in the slow-fast codimension 4 case. The reason we study this case is twofold. On one hand, the presence of the quartic term eliminates possibility of symmetric behavior of (2) and therefore simplifies our study, to some extent. On the other hand, we treat the codimension 4 case using a

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