ARTICLE IN PRESS YJDEQ:8715

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Application of an Adams type inequality to a two-chemical substances chemotaxis system

Kentarou Fujie [a,*], Takasi Senba [b]

[a] *Institute of Mathematics, Polish Academy of Sciences, Warsaw, 00-656, Poland*
[b] *Faculty of Science, Fukuoka University, Fukuoka, 814-0180, Japan*

Received 28 November 2016; revised 13 February 2017

Abstract

This paper deals with positive solutions of the fully parabolic system,

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla v) & \text{in } \Omega \times (0, \infty), \\ \tau_1 v_t = \Delta v - v + w & \text{in } \Omega \times (0, \infty), \\ \tau_2 w_t = \Delta w - w + u & \text{in } \Omega \times (0, \infty), \end{cases}$$

under homogeneous Neumann boundary conditions or mixed boundary conditions (no-flux and Dirichlet conditions) in a smooth bounded domain $\Omega \subset \mathbb{R}^n$ $(n \leq 4)$ with positive parameters $\tau_1, \tau_2, \chi > 0$ and nonnegative smooth initial data (u_0, v_0, w_0).

In the lower dimensional case $(n \leq 3)$, it is proved that for all reasonable initial data solutions of the system exist globally in time and remain bounded.

In the case $n = 4$, it is shown that in the radially symmetric setting solutions to the Neumann boundary value problem of the system exist globally in time and remain bounded if $\|u_0\|_{L^1(\Omega)} < (8\pi)^2/\chi$; as to the mixed boundary value problem, we will establish global existence and boundedness of solutions if $\|u_0\|_{L^1(\Omega)} < (8\pi)^2/\chi$ without radial symmetry.

The key ingredients are a Lyapunov functional and an Adams type inequality. A Lyapunov functional of the above problems will be constructed and the constant $(8\pi)^2/\chi$ is deduced from the critical constant in the Adams type inequality. This result is regarded as a generalization of the well-known 8π problem in the Keller–Segel system to higher dimensions.

* Corresponding author.
E-mail addresses: kentarou.fujie@gmail.com (K. Fujie), senba@fukuoka-u.ac.jp (T. Senba).

http://dx.doi.org/10.1016/j.jde.2017.02.031
0022-0396/

Please cite this article in press as: K. Fujie, T. Senba, Application of an Adams type inequality to a two-chemical substances chemotaxis system, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.02.031

MSC: 35B45; 35K45; 35Q92; 92C17

Keywords: Chemotaxis; Global existence; Lyapunov functional; Adams' inequality

Contents

1. Introduction

Our purpose is to establish global existence and boundedness of solutions to the following fully parabolic system:

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot (u \nabla v) & \text{in } \Omega \times (0, \infty), \\ \tau_1 v_t = \Delta v - v + w & \text{in } \Omega \times (0, \infty), \\ \tau_2 w_t = \Delta w - w + u & \text{in } \Omega \times (0, \infty), \end{cases} \tag{1.1}$$

in a bounded domain $\Omega \subset \mathbb{R}^n$ ($n \le 4$) with smooth boundary $\partial\Omega$, where the parameters τ_1, τ_2, and χ are positive. Suppose that one of the following boundary conditions:

$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = \frac{\partial w}{\partial \nu} = 0 \qquad \text{on } \partial\Omega \times (0, \infty) \tag{1.2}$$

or

$$\frac{\partial u}{\partial \nu} - \chi u \frac{\partial v}{\partial \nu} = v = w = 0 \qquad \text{on } \partial\Omega \times (0, \infty). \tag{1.3}$$

Moreover assume that

$$u(\cdot, 0) = u_0, \quad v(\cdot, 0) = v_0, \quad w(\cdot, 0) = w_0 \qquad \text{in } \Omega, \tag{1.4}$$

Please cite this article in press as: K. Fujie, T. Senba, Application of an Adams type inequality to a two-chemical substances chemotaxis system, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.02.031