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Boundedness and stabilization in a two-dimensional two-species chemotaxis-Navier–Stokes system with competitive kinetics

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Abstract

This paper deals with the two-species chemotaxis-Navier–Stokes system with Lotka–Volterra competitive kinetics

$$\begin{cases} (n_1)_t + u\cdot\nabla n_1 = \Delta n_1 - \chi_1\nabla\cdot(n_1\nabla c) + \mu_1 n_1(1-n_1-a_1n_2) & \text{in } \Omega\times(0,\infty),\\ (n_2)_t + u\cdot\nabla n_2 = \Delta n_2 - \chi_2\nabla\cdot(n_2\nabla c) + \mu_2 n_2(1-a_2n_1-n_2) & \text{in } \Omega\times(0,\infty),\\ c_t + u\cdot\nabla c = \Delta c - (\alpha n_1+\beta n_2)c & \text{in } \Omega\times(0,\infty),\\ u_t + (u\cdot\nabla)u = \Delta u + \nabla P + (\gamma n_1+\delta n_2)\nabla\phi, \quad \nabla\cdot u = 0 & \text{in } \Omega\times(0,\infty) \end{cases}$$

under homogeneous Neumann boundary conditions in a bounded domain $\Omega\subset\mathbb{R}^2$ with smooth boundary. This system consists of two models which were attracting mathematical interests. One is a chemotaxis-Navier–Stokes system which is known as a challenging model. The other is a two-species chemotaxis system with Lotka–Volterra competitive kinetics which has a complicated form. Both systems were recently well investigated; however, the above mixed system seems to be not studied yet. This paper gives results for global existence, boundedness and stabilization of solutions to the above system.

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1. Introduction and results

In this paper we consider the following two-species chemotaxis-Navier–Stokes system with Lotka–Volterra competitive kinetics:

$$\begin{cases} (n_1)_t + u\cdot\nabla n_1 = \Delta n_1 - \chi_1\nabla\cdot(n_1\nabla c) + \mu_1 n_1(1-n_1-a_1n_2) & \text{in } \Omega\times(0,\infty), \\ (n_2)_t + u\cdot\nabla n_2 = \Delta n_2 - \chi_2\nabla\cdot(n_2\nabla c) + \mu_2 n_2(1-a_2n_1-n_2) & \text{in } \Omega\times(0,\infty), \\ c_t + u\cdot\nabla c = \Delta c - (\alpha n_1+\beta n_2)c & \text{in } \Omega\times(0,\infty), \\ u_t + (u\cdot\nabla)u = \Delta u + \nabla P + (\gamma n_1+\delta n_2)\nabla\phi, \quad \nabla\cdot u = 0 & \text{in } \Omega\times(0,\infty), \\ \partial_\nu n_1 = \partial_\nu n_2 = \partial_\nu c = 0, \quad u = 0 & \text{on } \partial\Omega\times(0,\infty), \\ n_1(\cdot,0)=n_{1,0},\ n_2(\cdot,0)=n_{2,0},\ c(\cdot,0)=c_0,\ u(\cdot,0)=u_0 & \text{in } \Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^2$ with smooth boundary $\partial\Omega$ and ∂_ν denotes differentiation with respect to the outward normal of $\partial\Omega$; $\chi_1,\chi_2,a_1,a_2 \ge 0$ and $\mu_1,\mu_2,\alpha,\beta,\gamma,\delta > 0$ are constants; $n_{1,0},n_{2,0},c_0,u_0,\phi$ are known functions satisfying

$$0 < n_{1,0}, n_{2,0} \in C(\overline{\Omega}), \quad 0 < c_0 \in W^{1,q}(\Omega), \quad u_0 \in D(A^\theta), \tag{1.2}$$

$$\phi \in C^{1+\eta}(\overline{\Omega}) \tag{1.3}$$

for some $q > 2$, $\theta \in \left(\frac{1}{2}, 1\right)$, $\eta > 0$ and A is the Stokes operator.

The problem (1.1) is a model that describes the exercise of two species which react on a single chemoattractant in fluid, where n_1 and n_2 stand for densities of species, c shows the chemical concentration, u denotes the fluid velocity field and P represents the pressure of the fluid. The problem (1.1) comes from a problem on account of the influence of chemotaxis, the Lotka–Volterra kinetics and fluid. Here chemotaxis is the biased migration of cells toward more favourable environmental conditions, e.g., higher concentration of oxygen, which plays an outstanding role in a large range of biological applications [5]. Also, multi-species chemotaxis systems would be appearing in nature and were studied e.g. in [6] and [24]. In our model we focus on the case of two-species which react on single chemoattractant. Moreover, there are some observations which reveal dynamics concerning patterns and spontaneous emergence of turbulence in populations of aerobic bacteria suspended in sessile drops of water [19].

From a mathematical viewpoint it is fundamental to study whether global existence and behaviour of solutions to (1.1) can be clarified or not. Nowadays, we can find many successful studies about this basic question in some particular cases (see e.g., the recent survey [2]). In the case of single-species models reduced with $n_2 = 0$, Winkler [21] first attained global existence of solutions to (1.1) without Lotka–Volterra competitive kinetics in $\Omega \subset \mathbb{R}^2$ and also Winkler [22] obtained a precise stabilization result in the case that $N = 2$. When $N = 3$, Winkler [23] showed global existence of weak solutions to (1.1). On the other hand, when $-\alpha n_1 c$ in the third equation of (1.1) is replaced with $n_1 - c$, Tao and Winkler [18] established global existence and asymptotic behaviour of solutions to (1.1) with $\mu_1 > 0$ and $n_2 = 0$ in the case that $N = 2$, and

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