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Periodic solutions for a non-monotone family of delayed differential equations with applications to Nicholson systems

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Abstract

For a family of n-dimensional periodic delay differential equations which encompasses a broad set of models used in structured population dynamics, the existence of a positive periodic solution is obtained under very mild conditions. The proof uses the Schauder fixed point theorem and relies on the permanence of the system. A general criterion for the existence of a positive periodic solution for Nicholson's blowflies periodic systems (with both distributed and discrete time-varying delays) is derived as a simple application of our main result, generalizing the few existing results concerning multi-dimensional Nicholson models. In the case of a Nicholson system with discrete delays all multiples of the period, the global attractivity of the positive periodic solution is further analyzed, improving results in recent literature.

MSC: 34K13; 34K20; 92D25

Keywords: Delay differential equation; Periodic Nicholson system; Positive periodic solution; Schauder fixed point theorem; Permanence

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1. Introduction

In recent years, the question of the existence of periodic solutions for periodic delay differential equations (DDEs) has attracted the interest of many researchers, and a plethora of positive answers has been provided by using a variety of methods. To a large extent, the techniques used in the literature apply to a specific equation only, while other ones apply to a very particular class of DDEs, with emphasis on scalar models. For some classical models from mathematical biology, the available existence results require a very restrictive set of assumptions, which are not easily verifiable, much less extendable to other families of DDEs.

The main purpose of this paper is to investigate the existence of a positive periodic solution for a broad class of periodic and in general non-monotone n-dimensional DDEs which encompasses a large number of population models with patch structure. DDEs with patch structure have extensive applications in population dynamics, where the patch structure accounts for situations of heterogeneous environments due to several aspects, or in disease and epidemic models with different classes for cells or individuals, with transition among the classes. In particular, the study of periodic models is especially significant, as they reflect periodical variations of the weather or seasonality of the habitat in general, so the quest for positive periodic solutions for such models becomes quite relevant.

In this paper, we consider a family of periodic delayed population models with patch structure and multiple time-varying delays of the form

$$x_i'(t) = -d_i(t)x_i(t) + \sum_{j=1, j\neq i}^{n} a_{ij}(t)x_j(t) + \sum_{k=1}^{m} \beta_{ik}(t) \int_{t-\tau_{ik}(t)}^{t} b_{ik}(s, x_i(s))\, d_s\eta_{ik}(t,s),\ i = 1, \dots, n, \tag{1.1}$$

where all the coefficients and delay functions are assumed to be continuous, non-negative and periodic on t, with a common period $\omega > 0$, and $\eta_{ik}(t,s)$ are bounded, nondecreasing on s, locally integrable and ω-periodic on t. Some additional conditions on the coefficients $d_i(t), a_{ij}(t), \beta_{ik}(t)$ and on the nonlinearities $b_{ik}(t,x)$ will be assumed. Special attention will be given to the study of (1.1) with $\eta_{ik}(t,s) = H_{t-\tau_{ik}(t)}(s)$, where $H_t(s)$ is the Heaviside function $H_t(s) = 0$ if $s \le t$, $H_t(s) = 1$ if $s > t$. In this case, we obtain a system with discrete delays of the form

$$x_i'(t) = -d_i(t)x_i(t) + \sum_{j=1, j\neq i}^{n} a_{ij}(t)x_j(t) + \sum_{k=1}^{m} \beta_{ik}(t)h_{ik}(t, x_i(t - \tau_{ik}(t))),\ i = 1, \dots, n. \tag{1.2}$$

Many important delayed non-autonomous models from mathematical biology can be written in the form (1.1), see e.g. [15,20,25]. In Section 2, a descriptive set of hypotheses, as well as a brief biological interpretation of the model, will be given.

The present paper is a continuation of the research recently conducted by Faria, Obaya and Sanz in [7], where the asymptotic behavior of solutions for non-autonomous systems (1.2) was

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