ARTICLE IN PRESS YJDEQ:8725

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Resonances for Euler–Bernoulli operator on the half-line

Andrey Badanin *, Evgeny L. Korotyaev

Saint-Petersburg State University, Universitetskaya nab. 7/9, St. Petersburg, 199034 Russia

Received 2 October 2016

Abstract

We consider resonances for fourth order differential operators on the half-line with compactly supported coefficients. We determine asymptotics of a counting function of resonances in complex discs at large radius, describe the forbidden domain for resonances and obtain trace formulas in terms of resonances. We apply these results to the Euler–Bernoulli operator on the half-line. The coefficients of this operator are positive and constants outside a finite interval. We show that this operator does not have any eigenvalues and resonances iff its coefficients are constants on the whole half-line.

MSC: 34L25; 47E05; 47N50

Keywords: Resonances; Scattering; Fourth order operators

Contents

* Corresponding author.
E-mail addresses: an.badanin@gmail.com, a.badanin@spbu.ru (A. Badanin), korotyaev@gmail.com, e.korotyaev@spbu.ru (E.L. Korotyaev).

http://dx.doi.org/10.1016/j.jde.2017.02.041

Please cite this article in press as: A. Badanin, E.L. Korotyaev, Resonances for Euler–Bernoulli operator on the half-line, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.02.041

1. Introduction and main results

1.1. Introduction

There are many results about Schrödinger operators $\mathbf{H} = -\Delta + V(\mathbf{x}), \mathbf{x} \in \mathbb{R}^3$ with compactly supported potentials V on $L^2(\mathbb{R}^3)$, see [38,35] and references therein. In the important physical case the potential $V(|\mathbf{x}|)$ is symmetric and depends only on radius $r = |\mathbf{x}| > 0$. The standard transform $y(\mathbf{x}) \mapsto ry(\mathbf{x})$ and expansion in spherical harmonics give that $\mathbf{H}$ is unitarily equivalent to a direct sum of the Schrödinger operators acting on $L^2(\mathbb{R}_+)$. The first operator from this sum is given by $-\frac{\partial^2}{\partial r^2} + V(r)$. There are a lot of results about the resonances for 1-dimensional case, see [9,18,33,37] and references therein.

Now we consider a biharmonic type operator $\mathbf{B} = \frac{1}{\beta(r)}\Delta\alpha(r)\Delta$ on $\mathbb{R}^3$, where $\alpha(r), \beta(r)$ are some positive functions depending on the radius r only. The similar operators on $\mathbb{R}^2$ describe, for example, vibrations of plates with an axisymmetric variable thickness, see [25]. The separation of variables (similar to the case of Schrödinger operators), show that the operator $\mathbf{B}$ is unitarily equivalent to a direct sum of fourth order operators acting on $L^2(\mathbb{R}_+)$. The first operator from this sum is given by an Euler-Bernoulli operator $\frac{1}{b}\frac{d^2}{dr^2}a\frac{d^2}{dr^2}$ on the half-line with some positive coefficients a, b. Remark that the Euler–Bernoulli operators are related with the problems of vibrations of beams, see [36].

The standard unitary Liouville type transformation reduces the Euler–Bernoulli operator into a fourth order operator H on the half-line defined by (1.1) with some coefficients p, q. Thus in order to discuss resonances for Euler–Bernoulli operators we consider a fourth order operators H acting on $L^2(\mathbb{R}_+)$ and given by

$$Hy = H_0 y + Vy, \quad H_0 = \partial^4, \qquad V = 2\partial p\partial + q, \tag{1.1}$$

with the boundary conditions

$$y(0) = y''(0) = 0, \tag{1.2}$$

where $\partial = \frac{d}{dx}$. We consider the operator $H_0 = \partial^4$ as unperturbed and the operator V is its perturbation. The coefficients p, q are compactly supported and belong to the space $\mathcal{H}_0$, where $\mathcal{H}_m = \mathcal{H}_m(\gamma), m = 0, 1, 2, \dots$, is the spaces of functions defined by

$$\mathcal{H}_m = \left\{ f \in L^1_{real}(\mathbb{R}_+) : \operatorname{supp} f \in [0, \gamma],\ f^{(m)} \in L^1(0, \gamma) \right\}$$

for some $\gamma > 0$. The boundary conditions (1.2) are taken for reasons of convenience. The operators with other boundary conditions can be considered similarly.

It is well known that the operator H is self-adjoint and is defined on the corresponding form domain, see Sect. 2.4. It has purely absolutely continuous spectrum $[0, \infty)$ plus a finite number of simple real eigenvalues, see Proposition 1.1.

Please cite this article in press as: A. Badanin, E.L. Korotyaev, Resonances for Euler–Bernoulli operator on the half-line, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.02.041