ARTICLE IN PRESS YJDEQ:8835

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J. Differential Equations ●●● (●●●●) ●●●–●●●

Journal of Differential Equations

www.elsevier.com/locate/jde

Singularly perturbed critical Choquard equations ☆

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Received 15 November 2016; revised 18 April 2017

Abstract

In this paper we study the semiclassical limit for the singularly perturbed Choquard equation

$$-\varepsilon^2 \Delta u + V(x)u = \varepsilon^{\mu-3}\Big(\int_{\mathbb{R}^3} \frac{Q(y)G(u(y))}{|x-y|^{\mu}}dy\Big)Q(x)g(u) \quad \text{in } \mathbb{R}^3,$$

where $0 < \mu < 3$, ε is a positive parameter, V, Q are two continuous real function on $\mathbb{R}^3$ and G is the primitive of g which is of critical growth due to the Hardy–Littlewood–Sobolev inequality. Under suitable assumptions on g, we first establish the existence of ground states for the critical Choquard equation with constant coefficients. Next we establish existence and multiplicity of semi-classical solutions and characterize the concentration behavior by variational methods.

MSC: 35J20; 35J60; 35B33

Keywords: Choquard equation; Critical growth; Hardy–Littlewood–Sobolev inequality; Semi-classical solutions

☆ Claudianor O. Alves is partially supported by CNPq/Brazil 304036/2013-7; Marco Squassina is member of the Gruppo Nazionale per l'Analisi Matematica, la Probabilità e le loro Applicazioni; Minbo Yang is partially supported by NSFC (11571317, 11671364) and ZJNSF (LY15A010010).

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http://dx.doi.org/10.1016/j.jde.2017.05.009
0022-0396/

Please cite this article in press as: C.O. Alves et al., Singularly perturbed critical Choquard equations, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.05.009

Contents

1. Introduction and results

The stationary Choquard equation

$$-\Delta u+V(x)u=\Big(\int\limits_{\mathbb{R}^N}\frac{|u(y)|^p}{|x-y|^\mu}dy\Big)|u|^{p-2}u,\qquad \text{in } \mathbb{R}^N,$$

where $N\geq 3$, $0<\mu<N$, arises in many interesting physical situations in quantum theory and plays an important role in the theory of Bose–Einstein condensation where it accounts for the finite-range many-body interactions. For $N=3$, $p=2$ and $\mu=1$, it was investigated by Pekar in [29] to study the quantum theory of a polaron at rest. In [19], Choquard applied it as approximation to Hartree–Fock theory of one-component plasma. This equation was also proposed by Penrose in [23] as a model of selfgravitating matter and is known in that context as the Schrödinger–Newton equation. For a complete and updated discussion upon the current literature of such problems, we refer the interested reader to the guide [28]. We also mention [14], where the fractional case is treated.

In the present paper we are interested in the existence, multiplicity and concentration behavior of the semi-classical solutions of the singularly perturbed nonlocal elliptic equation

$$-\varepsilon^2\Delta u+V(x)u=\varepsilon^{\mu-3}\Big(\int\limits_{\mathbb{R}^3}\frac{Q(y)G(u(y))}{|x-y|^\mu}dy\Big)Q(x)g(u),\qquad \text{in } \mathbb{R}^3, \tag{1.1}$$

where $0<\mu<3$, ε is a positive parameter, V, Q are real continuous functions on $\mathbb{R}^3$. As ε goes to zero in (1.1), the existence and asymptotic behavior of the solutions of the singularly perturbed equation (1.1) is known as the *semi-classical problem*. It was used to describe the transition between of Quantum Mechanics and classical Mechanics. For the local Schrödinger equation

$$-\varepsilon^2\Delta u+V(x)u=g(u)\quad \text{in } \mathbb{R}^N,$$

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