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On an extended second Painlevé hierarchy

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Abstract

We present a new extension of the second Painlevé hierarchy and study its properties. In addition to Lax pairs, Bäcklund transformations, auto-Bäcklund transformations and basic special integrals, we also consider a new phenomenon whereby we obtain relations between systems of different orders but of the same form. The extension made here of the second Painlevé hierarchy is based on the use of non-isospectral scattering problems and so is quite general. We thus expect to be able to obtain similar extensions of other Painlevé hierarchies, including not only for continuous examples but also for discrete and differential-delay examples. We believe that our work is also of relevance for Painlevé classification, since it gives information about classes of equation that may be of interest and in addition provides a key to the possible identification of equations isolated in such a process.
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1. Introduction

Over the last twenty years or so there has been a great deal of interest in the derivation of Painlevé hierarchies and the study of their properties. The aim of the present paper is to present and study a new extended second Painlevé (P_{II}) hierarchy. The extension made here stems from the use of non-isospectral scattering problems and can clearly also readily be made for

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discrete and differential-delay Painlevé hierarchies. We thus claim that our work here represents a new way forward in the derivation of extended versions of Painlevé equations and hierarchies, whether they be continuous, discrete or differential-delay. In this sense we also believe that our work has important ramifications for Painlevé classification, again for these various kinds of equation.

We note that the first papers to consider Painlevé hierarchies were in fact published some twenty years before the above-mentioned surge in interest. Thus Ablowitz and Segur [1] and Airault [2], making use respectively of the modified Korteweg–de Vries (mKdV) hierarchy or of this last and also the Korteweg–de Vries (KdV) hierarchy, defined using similarity reduction a hierarchy of ordinary differential equations (ODEs) having as first member the second Painlevé equation,

$$\psi[v]\tilde{\mathcal{R}}^{n-1}[v]v_x + 2h_1 x v - \alpha_n = 0, \qquad n = 1, 2, \ldots, \tag{1.1}$$

where

$$\psi[v] = \partial_x - 4v\partial_x^{-1}v, \qquad \tilde{\mathcal{R}}[v] = \partial_x \psi[v] = \partial_x \left(\partial_x - 4v\partial_x^{-1}v\right) \tag{1.2}$$

is the recursion operator of the mKdV hierarchy, h_1 is a nonzero constant and α_n is an arbitrary constant. Without loss of generality we may take $h_1 = -1/2$, in which case (1.1) for $n = 1$ gives the second Painlevé equation as usually written, i.e.,

$$v_{xx} = 2v^3 + xv + \alpha_1. \tag{1.3}$$

Airault [2] also obtained auto-Bäcklund transformations (auto-BTs) for the hierarchy (1.1). This hierarchy was subsequently rederived in [3], along with a first Painlevé (P_I) hierarchy. An alternative form of the auto-BTs of (1.1) was given in [4].

Additional terms derived from lower order mKdV flows can be added to the hierarchy (1.1) to give [5,6]

$$\psi[v]\sum_{k=1}^{n} c_k \tilde{\mathcal{R}}^{k-1}[v]v_x + c_0 v + 2h_1 x v - \alpha_n = 0, \qquad n = 1, 2, \ldots, \tag{1.4}$$

where all c_k, $k = 0, 1, \ldots, n$, are constant (and h_1 is a nonzero constant and α_n is an arbitrary constant). In the present paper we will use the terminology we have adopted in previous papers and refer to Painlevé hierarchies where, amongst all possible terms corresponding to flows of the related completely integrable partial differential equation (PDE) hierarchy, only terms corresponding to the highest weight flow are present, as standard Painlevé hierarchies. We will refer to Painlevé hierarchies where terms corresponding to lower weight flows of the related completely integrable PDE hierarchy are also present as generalized Painlevé hierarchies. Thus (1.1) will be referred to as the standard P_{II} hierarchy and (1.4) as the generalized P_{II} hierarchy.

Whilst the use of similarity reduction might be the most obvious means of deriving Painlevé hierarchies, it is not of course the only technique available. In [7] the use of non-isospectral scattering problems was proposed as a means of deriving Painlevé equations, with the first and second Painlevé equations being given as examples; this approach was then used (and also further extended) in [8] to derive Painlevé hierarchies. It might even be argued that the use of

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