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Higher order elliptic operators on variable domains. Stability results and boundary oscillations for intermediate problems

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Abstract

We study the spectral behavior of higher order elliptic operators upon domain perturbation. We prove general spectral stability results for Dirichlet, Neumann and intermediate boundary conditions. Moreover, we consider the case of the bi-harmonic operator with those intermediate boundary conditions which appears in the study of hinged plates. In this case, we analyze the spectral behavior when the boundary of the domain is subject to a periodic oscillatory perturbation. We will show that there is a critical oscillatory behavior and the limit problem depends on whether we are above, below or just sitting on this critical value. In particular, in the critical case we identify the strange term appearing in the limiting boundary conditions by using the unfolding method from homogenization theory.

MSC: 35J40; 35B20; 35B27; 35P15

Keywords: Higher order elliptic operators; Dirichlet; Neumann; Intermediate boundary conditions; Oscillatory boundaries; Homogenization

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1. Introduction

In this paper, we consider the general problem of the spectral behavior of an elliptic partial differential operator (i.e., the behavior of its eigenvalues, eigenfunctions as well as of the solutions to the corresponding Poisson problem) when the underlying domain is perturbed. In $\mathbb{R}^N$ with $N \geq 2$, we will consider a family of domains $\{\Omega_\epsilon\}_{0<\epsilon\leq\epsilon_0}$ which approach a limiting domain Ω as $\epsilon \to 0$, in certain sense to be specified and we will also consider higher order selfadjoint operators (order $2m$ with $m \geq 1$) with not necessarily constant coefficients and with certain boundary conditions. The operators will have compact resolvent and therefore the spectrum will consist only of eigenvalues of finite multiplicity.

Importantly, the associated energy spaces, generically denoted by $V(\Omega_\epsilon)$, will satisfy the condition $W_0^{m,2}(\Omega_\epsilon) \subset V(\Omega_\epsilon) \subset W^{m,2}(\Omega_\epsilon)$ and will depend on the domain and the boundary conditions considered. We will consider different types of boundary conditions according to the choice of the spaces $V(\Omega_\epsilon)$. If $V(\Omega_\epsilon) = W_0^{m,2}(\Omega_\epsilon)$ they will be called Dirichlet boundary conditions, if $V(\Omega_\epsilon) = W^{m,2}(\Omega_\epsilon)$ they will be called Neumann boundary conditions, and in case $V(\Omega_\epsilon) = W^{m,2}(\Omega_\epsilon) \cap W_0^{k,2}(\Omega_\epsilon)$ for certain $1 \leq k \leq m-1$, they will be called "intermediate boundary condition". We refer to [7] and references therein for a pioneer discussion on the stability properties under the three types of boundary conditions, including an analysis of the so-called Babuška–Sapondzhyan paradox. We also refer to [26] for a further discussion on the paradox and [27] for a general reference in this type of problems. We mention that sharp stability estimates for the eigenvalues of higher order operators subject to Dirichlet and Neumann boundary conditions have been recently proved in [12] where uniform classes of domain perturbations have been considered (see also [13,14] for related results); moreover, in [9,10] further restrictions on the classes of open sets allow obtaining also analyticity results.

Our goal is twofold. On one hand, we will provide a rather general condition describing the way the domains converge to the limiting one, which will guarantee the spectral convergence of the operator in Ω_ϵ to the appropriate limiting operator in Ω. The condition, which we will denote by (C), see Section 3 below, is expressed intrinsically and it is posed independently of the boundary conditions that we consider. Needless to say that for a particular family of perturbed domains to check that the condition is satisfied will depend heavily on the boundary conditions imposed. This condition generalizes previous ones formulated for Dirichlet and Neumannn boundary conditions.

On the other hand, we will focus on the case of higher order operators with "intermediate boundary" conditions, paying special attention to the case of the biharmonic operator. We will obtain almost sharp conditions on the way the boundaries can be perturbed to guarantee the spectral convergence with preservation of the same intermediate boundary conditions for general higher order operators. Afterwards we will analyze in detail the case of the biharmonic when the boundary of the domain presents an oscillatory behavior. We will see that there is a critical oscillatory behavior such that, when the oscillations are below this threshold we have spectral stability, while for oscillations above this value we approach a problem with Dirichlet boundary conditions. For exactly the threshold value, there appears an extra term in the boundary condition for the limiting problem, which maybe interpreted as a "strange curvature". The existence of this critical value is well known in other situations. See for instance the seminal paper [20] and also, [26]. In other context see [17,2,25].

We describe now the contents of the paper. In Section 2 we set up the operators, fix the notation and include a subsection where we describe the basic elements of the technique called "compact convergence of operators" which will be used in this paper. In Section 3 we state con-

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