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Global existence and exponential stability for the compressible Navier–Stokes equations with discontinuous data

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Abstract

The initial boundary value problem for the compressible barotropic Navier–Stokes equations is investigated in the case that the initial density has a jump discontinuity across an interior closed curve in two-dimensional bounded domain. If the initial data is a small perturbation of the constant state and the interior closed curve is near a circle inside the domain, the global existence and large time behavior of the piecewise strong solution is shown, in particular, the jump of the fluid density across the convecting curve decays exponentially in time.

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Keywords: Compressible Navier–Stokes equations; Global existence; Large time behavior; Stability; Discontinuous

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1. Introduction

In this paper, we consider the compressible isentropic Navier–Stokes equations in bounded domain $B_{R_0} \subset \mathbb{R}^2$:

$$\partial_t \rho + div(\rho u) = 0 \qquad \text{in } B_{R_0}, \tag{1.1}$$

$$\partial_t(\rho u) + div(\rho u \otimes u) - div\mathcal{T} = 0 \qquad \text{in } B_{R_0} \tag{1.2}$$

with the following boundary condition and the initial data

$$u = 0 \qquad \text{on } \partial B_{R_0}, \tag{1.3}$$

$$(\rho, u)|_{t=0} = (\rho_0, u_0) \qquad \text{in } B_{R_0}, \tag{1.4}$$

where $\mathcal{T} = -p(\rho)I + \mu D(u) + (\nu - \mu)divuI$, $D(u)_{ij} = \partial_i u_j + \partial_j u_i$, and μ and ν are two positive constants. We consider the case that the initial density ρ_0 is piecewise smooth with a jump discontinuity across some interior closed curve Γ_0 in B_{R_0}, which can be described as

$$\Gamma_0 := \left\{ R^0(\theta) \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \;\middle|\; \theta \in [0, 2\pi) \text{ and } |R^0|_{L^\infty([0,2\pi))} < R_0 \right\}.$$

As the time evolutes, the convecting curve $\Gamma(t)$ is described as

$$\Gamma(t) := \left\{ x \in \mathbb{R}^2 \middle| x = R(t,\theta) \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix},\ \theta \in [0, 2\pi) \right\},$$

and across the curve $\Gamma(t)$ the solution (ρ, u, R) satisfies the Rankine–Hugoniot condition

$$u_+ = u_- \qquad \text{on } \Gamma(t), \tag{1.5}$$

$$(\mathcal{T}_+ - \mathcal{T}_-)\vec{n} = 0 \qquad \text{on } \Gamma(t), \tag{1.6}$$

$$\partial_t R + \frac{\partial_\theta R}{R} u \cdot \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} = u \cdot \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \qquad \text{on } \Gamma(t), \tag{1.7}$$

$$R|_{t=0} = R^0,$$

where $\vec{n}$ denotes the unit outward normal vector of domain $\Omega_-(t)$

$$\vec{n} = \frac{1}{\sqrt{R^2 + |\partial_\theta R|^2}} \left[R \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} - \partial_\theta R \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \right],$$

and $\Omega_+(t) := B_{R_0} \setminus \Omega_-(t)$ (referring to Fig. 1). By the continuity equation (1.1) and the kinematic condition (1.7), the total mass $M_\pm$ is conserved respectively in domain $\Omega_\pm(t)$, i.e.

$$\int_{\Omega_\pm(t)} \rho_\pm(x,t)dx = \int_{\Omega_\pm(0)} \rho_{0,\pm}(x)dx := M_\pm. \tag{1.8}$$

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