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Unknown input observer design and output feedback stabilization for multi-dimensional wave equation with boundary control matched uncertainty ☆

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Abstract

In this paper, we consider boundary output feedback stabilization for a multi-dimensional wave equation with boundary control matched unknown nonlinear internal uncertainty and external disturbance. A new unknown input type extended state observer is proposed to recover both state and total disturbance which consists of internal uncertainty and external disturbance. A key feature of the proposed observer in this paper is that we do not use the high-gain to estimate the disturbance. By the active disturbance rejection control (ADRC) strategy, the total disturbance is compensated (canceled) in the feedback loop, which together with a collocated stabilizing controller without uncertainty, leads to an output feedback stabilizing feedback control. It is shown that the resulting closed-loop system is well-posed and asymptotically stable under weak assumption on internal uncertainty and external disturbance. The numerical experiments are carried out to show the effectiveness of the proposed scheme.

MSC: 37L15; 93D15; 93B51; 93B52

Keywords: Wave equation; Boundary control; Active disturbance rejection control; Disturbance rejection; Output feedback stabilization

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1. Introduction

Since the robustness concept was introduced to control theory from the early 1970s ([18]), the capability of dealing with uncertainty has become one of the most important performances for control systems. Many control methods have been developed to cope with internal uncertainty and external disturbance. These include sliding mode control for various uncertainties ([20]); internal model principle for output regulation; adaptive control for unknown parameter identification; and robust control ([3]) which is a remarkable paradigm shift in modern control theory for general plant uncertainty. Most of these methods, among many others, focus however, on the worst case scenario which makes the controller design rather conservative. The worth mentioning methods are internal model principle ([4,19]) and adaptive control ([1,11]) where the uncertainty is estimated and compensated. The estimation/cancellation strategy is an economic way in dealing with uncertainty. In this regard, an emerging control technology named active disturbance rejection control (ADRC) is an epitomized approach to cope with vast uncertainty in control systems ([10]). The uncertainties dealt with by ADRC are much more complicated. For instance, ADRC can deal with the coupling between the external disturbances, the system unmodeled dynamics, and the superadded unknown part of control input. The most remarkable feature of ADRC is that the disturbance is estimated, in real time, through an extended state observer and is canceled in the feedback loop. This reduces the control energy significantly in practice [24]. In the past two decades, there are numerous researches on ADRC from perspectives of both engineering and mathematics. Very recently, we applied ADRC to stabilization for multi-dimensional wave equation with external disturbance in [8] where the full state feedback control was used and a high gain extended state was adopted. The output feedback control for PDEs by ADRC is much complicated. In paper [5], an unknown input observer was designed by variable structure control first and then ADRC was applied to design an observer-based feedback control for 1-d wave equation. But the observer in [5] is very complicated with discontinuous injection of the output and hence is hard to generalize to other PDEs.

The aim of this paper is to design a new extended state observer ([6]) in terms of measurements from the boundary and an interior domain which can be of arbitrarily small measure; and an extended state observer based output feedback stabilizing control law for a multi-dimensional wave equation subject to general internal uncertainty and external boundary disturbance. The uncertainty is hence more general than that considered in [8], and we do not use the high-gain to estimate the state and total disturbance as that in [5,8] to avoid possible peaking value problem.

The system that we are concerned with is a multi-dimensional wave equation with Neumann boundary control and unknown nonlinear internal uncertainty and external disturbance, governed by the following partial differential equation:

$$\begin{cases} w_{tt}(x,t) = \Delta w(x,t),\ x \in \Omega,\ t > 0, \\ w(x,t)|_{\Gamma_0} = 0,\ x \in \Gamma_0,\ t \geq 0, \\ \dfrac{\partial w(x,t)}{\partial \nu}\Big|_{\Gamma_1} = f(w(\cdot,t)) + d(x,t) + v(x,t),\ x \in \Gamma_1,\ t \geq 0, \\ w(x,0) = w_0(x),\ w_t(x,0) = w_1(x),\ x \in \Omega, \\ y(x,t) = (w(x,t)|_{\Gamma_1}, w_t(x,t)|_{\Gamma_1}, w_t(x,t)|_{\omega}),\ t \geq 0, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^n (n \geq 2)$ is an open bounded domain with a smooth C^2-boundary $\Gamma = \overline{\Gamma_0} \cup \overline{\Gamma_1}$ where Γ_0 and Γ_1 are subsets of Γ, $\text{int}(\Gamma_0) \neq \emptyset$, $\text{int}(\Gamma_1) \neq \emptyset$, $\Gamma_0 \cap \Gamma_1 = \emptyset$; ν is the unit normal

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