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Dynamics and asymptotic profiles of steady states of an epidemic model in advective environments ☆

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Abstract

We study the dynamics of a SIS epidemic model of reaction–diffusion–advection type. The persistence of infected and susceptible populations and the global stability of the disease free equilibrium are established when the basic reproduction number is greater than or less than or equal to one, respectively. We further consider the effects of diffusion and advection on asymptotic profiles of endemic equilibrium: When the advection rate is relatively large comparing to the diffusion rates of both populations, then two populations persist and concentrate at the downstream end. As the diffusion rate of the susceptible population tends to zero, the density of the infected population decays exponentially for positive advection rate but linearly when there is no advection. Our results suggest that advection can help speed up the elimination of disease.

MSC: 35J55; 35B32

Keywords: Disease dynamics; Advective environment; Spatial heterogeneity; Asymptotic profile; Steady state

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1. Introduction

It has been widely recognized that environmental heterogeneity and individual motility are significant factors that should be taken into account in studying disease dynamics. For the Susceptible-Infected-Susceptible (SIS) epidemic reaction–diffusion models, some recent works are able to capture the effect of the spatial heterogeneity of environment and movement of individuals on the persistence or extinction of diseases [2,3,7,9,10,13,20–23,28]. In [2] Allen et al. proposed a SIS epidemic reaction–diffusion model without advection. In [21] Peng and Liu discussed the global stability of the endemic equilibrium in some special cases for the model of Allen et al. The effects of large and small diffusion rates of the susceptible and infected populations on the persistence and extinction of the disease were considered in [20,22]. Allen et al. also investigated a discrete SIS-model in [1]. Peng and Zhao [23] recently considered the same SIS reaction–diffusion model of Allen et al., but the rates of disease transmission and recovery are assumed to be spatially heterogeneous and temporally periodic. In [7,28] the authors consider an SIS model with mass action infection mechanism. In [13] Li et al. provided qualitative analysis on an SIS reaction diffusion system with a linear source term. Ge et al. introduced a free boundary model for characterizing the spreading front of the disease in [9]. In these works the populations are assumed to adopt random diffusion in the habitats.

In some heterogeneous environments populations may assume biased or passive movement in certain directions [4,19], e.g., due to the external environmental forces such as wind [6], water flow [12,15–18] and so on, which usually can be described by adding an advection term to the existing reaction–diffusion models. For the spatial epidemic model with advection in heterogeneous environment, it is of interest to understand how the diffusion and advection jointly affect the persistence or extinction of the infectious diseases. Such studies may have significant implications for predicting the patterns of disease occurrence and for designing optimal control strategies as well.

The following SIS reaction–diffusion–advection model in one dimensional domain was considered in [5]:

$$\begin{cases} \bar{S}_t = d_S \bar{S}_{xx} - q\bar{S}_x - \beta(x)\dfrac{\bar{S}\bar{I}}{\bar{S}+\bar{I}} + \gamma(x)\bar{I}, & 0 < x < L,\ t > 0, \\ \bar{I}_t = d_I \bar{I}_{xx} - q\bar{I}_x + \beta(x)\dfrac{\bar{S}\bar{I}}{\bar{S}+\bar{I}} - \gamma(x)\bar{I}, & 0 < x < L,\ t > 0, \\ d_S\bar{S}_x - q\bar{S} = d_I\bar{I}_x - q\bar{I} = 0, & x = 0, L,\ t > 0, \\ \bar{S}(x,0) = \bar{S}_0(x) \ge 0, \quad \bar{I}(x,0) = \bar{I}_0(x) \ge 0, & 0 < x < L, \end{cases} \tag{1.1}$$

where $\bar{S}(x,t)$ and $\bar{I}(x,t)$ denote the density of susceptible and infected individuals at time t and position x in the interval $[0, L]$, respectively; the positive constants d_S and d_I are diffusion coefficients for the susceptible and infected populations; q is the effective speed of the current (sometimes we call q the advection rate); L is the size of the habitat, and we call $x = 0$ the upstream end and $x = L$ the downstream end. The functions $\beta(x)$ and $\gamma(x)$ are assumed to be positive, Hölder continuous on $[0, L]$ and they represent the rates of disease transmission and recovery at x, respectively. Here both populations satisfy no-flux boundary conditions, which means that there is no population flux across the upstream and downstream ends, so that both susceptible and infected populations live in a closed environment. As the term $\bar{S}\bar{I}/(\bar{S}+\bar{I})$ is a Lipschitz continuous function of $\bar{S}$ and $\bar{I}$ in the open first quadrant, its definition can be extended