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Existence of quasiperiodic solutions of elliptic equations on $\mathbb{R}^{N+1}$ via center manifold and KAM theorems

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Abstract

We consider elliptic equations on $\mathbb{R}^{N+1}$ of the form

$$\Delta_x u + u_{yy} + g(x,u) = 0, \quad (x,y) \in \mathbb{R}^N \times \mathbb{R}, \tag{1}$$

where $g(x,u)$ is a sufficiently regular function with $g(\cdot,0) \equiv 0$. We give sufficient conditions for the existence of solutions of (1) which are quasiperiodic in y and decaying as $|x| \to \infty$ uniformly in y. Such solutions are found using a center manifold reduction and results from the KAM theory. We discuss several classes of nonlinearities g to which our results apply.

MSC: 35B08; 35B15; 35J61; 37J40; 37K55; 35J10

Keywords: Elliptic equations; Entire solutions; Quasiperiodic solutions; Center manifold; KAM theorem; Nemytskii operators on Sobolev spaces

Contents

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1. Introduction

In this paper, we consider elliptic equations of the form

$$\Delta u + u_{yy} + g(x,u) = 0, \quad (x,y) \in \mathbb{R}^N \times \mathbb{R}, \tag{1.1}$$

where $(x,y) \in \mathbb{R}^N \times \mathbb{R}$, Δ is the Laplacian in x, and $g : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is a sufficiently smooth function satisfying $g(\cdot,0) \equiv 0$. We investigate solutions of (1.1) which decay to 0 as $|x| \to \infty$, uniformly in y. Our concern is the behavior of such solutions in the remaining variable y; specifically, we are interested in the existence of solutions which are quasiperiodic in y. The purpose of this article is twofold. First, we build a general framework for studying solutions of (1.1) using tools from dynamical systems, such as the center manifold theorem and the Kolmogorov–Arnold–Moser (KAM) theory. Then we show how these techniques yield quasiperiodic solutions in some specific classes of equations.

Geometric properties of solutions of (1.1) have been extensively studied by many authors. Best understood are positive solutions which decay to 0 in all variables. If g satisfies suitable assumptions, involving in particular symmetry and monotonicity conditions with respect to x, then a classical result of [30] establishes reflectional symmetry of such solutions, or even the radial symmetry about some origin in $\mathbb{R}^{N+1}$ if g is independent of x (see also [11–13,25,43,44] or the surveys [10,51,55] for related symmetry results and additional references). It is very likely, and has already been proved in some situations, that, under similar hypotheses on g, bounded positive solutions which decay as $|x| \to \infty$ uniformly in y, but do not necessarily decay in y, enjoy the symmetry in x (see [33] for results of this form). Several authors have also exposed complexities of various solutions which do not decay at infinity. Examples, with $g = g(u)$, include multi-bump solutions decaying along all but finitely many rays [45], saddle shaped solutions and general multiple-end solutions [22,23,40], as well as solutions having both fronts (transitions) and bumps [62].

Solutions of the form considered in the present paper (that is, solutions decaying in x uniformly in y) were examined by Dancer in [18]. Considering homogeneous nonlinearities

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