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J. Differential Equations 263 (2017) 1843–1873

Journal of Differential Equations

www.elsevier.com/locate/jde

Thermal equilibrium solution to new model of bipolar hybrid quantum hydrodynamics

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Received 11 September 2016

Available online 31 March 2017

Abstract

In this paper we study the hybrid quantum hydrodynamic model for nano-sized bipolar semiconductor devices in thermal equilibrium. By introducing a hybrid version of the Bhom potential, we derive a bipolar hybrid quantum hydrodynamic model, which is able to account for quantum effects in a localized region of the device for both electrons and holes. Coupled with Poisson equation for the electric potential, the steady-state system is regionally degenerate in its ellipticity, due to the quantum effect only in part of the device. This regional degeneracy of ellipticity makes the study more challenging. The main purpose of the paper is to investigate the existence and uniqueness of the weak solutions to this new type of equations. We first establish the uniform boundedness of the smooth solutions to the modified bipolar quantum hydrodynamic model by the variational method, then we use the compactness technique to prove the existence of weak solutions to the original hybrid system by taking hybrid limit. In particular, we account for two different kinds of hybrid behaviour. We perform the first hybrid limit when both electrons and holes behave quantum in a given region of the device, and the second one when only one carrier exhibits hybrid behaviour, whereas the other one is presented classically in the whole domain. The semi-classical limit results are also obtained. Finally, the theoretical results are tested numerically on a simple toy model.

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http://dx.doi.org/10.1016/j.jde.2017.03.032

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1. Introduction and derivation of the model

Due to recent progresses in semiconductor technology, we are able to project and produce nano-sized devices, operated by means of quantum effects. Quantum hydrodynamic models (QHDs), which describe such devices, give a fairly accurate account of the macroscopic behaviour of ultra small semiconductor devices only in terms of macroscopic quantities such as particle densities, current densities and electric fields (see [1,2,18,21,22,24,23,26,27,30,28] and reference therein). The bipolar QHD, reduced in thermal equilibrium, is the following 3×3 system of stationary equations [30,31]

$$\begin{cases} n\nabla V + T_n \nabla W_n(n) - 2\varepsilon^2 n \nabla\left(\frac{\Delta\sqrt{n}}{\sqrt{n}}\right) = 0, \\ -p\nabla V + T_p \nabla W_p(p) - 2\varepsilon^2 p\xi \nabla\left(\frac{\Delta\sqrt{p}}{\sqrt{p}}\right) = 0, \\ -\lambda^2 \Delta V = n - p - C, \\ \int_\Omega n(x)dx = N, \quad \int_\Omega p(x)dx = P, \quad \int_\Omega V(x)dx = 0, \end{cases} \tag{1}$$

where the unknown functions $n(x) \geq 0$, $p(x) \geq 0$ and $V(x)$ represent the particle density of electrons in the conduction band, the particle density of holes in the valence band, and the electrostatic potential, respectively. Ω is a bounded domain in $\mathbb{R}^d$, $d = 1, 2, 3$. ε is the scaled Planck's constant, and ξ is the ratio of the effective masses of electrons and holes. Without loss of generality, we assume $\xi = 1$ throughout the paper. T_n and T_p are the temperature constants for the electrons and the holes, respectively, and the constant λ is the minimal Debye length. $W_n(n)$ and $W_p(p)$ are the pressure functions for the electrons and the holes, respectively, and both are positive, continuously differentiable and increasing. $C(x)$ is the doping profile, which is assumed to be equal to $N_D - N_A$, where $N_D = N_D(x) \geq 0$ and $N_A = N_A(x) \geq 0$ are the space densities of donor and acceptor atoms, respectively. N (correspondingly P) is the total numbers of electrons (holes) in the conductivity band (the valence band), given by

$$N = n_i + \int_\Omega N_D(x)dx, \quad P = n_i + \int_\Omega N_A(x)dx,$$

where $n_i > 0$ is an intrinsic constant taking into account that the number of electrons (holes) in the conduction (valence) band is not only determined by doping but also by intrinsic thermal