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Spreading in space–time periodic media governed by a monostable equation with free boundaries, Part 1: Continuous initial functions ☆

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Abstract

We aim to classify the long-time behavior of the solution to a free boundary problem with monostable reaction term in space–time periodic media. Such a model may be used to describe the spreading of a new or invasive species, with the free boundary representing the expanding front. In time-periodic and space homogeneous environment, as well as in space-periodic and time autonomous environment, such a problem has been studied recently in [11,12]. In both cases, a spreading–vanishing dichotomy has been established, and when spreading happens, the asymptotic spreading speed is proved to exist by making use of the corresponding semi-wave solutions. The approaches in [11,12] seem difficult to apply to the current situation where the environment is periodic in both space and time. Here we take a different approach, based on the methods developed by Weinberger [31,32] and others [16,22–24,26], which yield the existence of the spreading speed without using traveling wave solutions. In Part 1 of this work, we establish the existence and uniqueness of classical solutions for the free boundary problem with continuous initial data, extending the existing theory which was established only for C^2 initial data. This will enable us to develop Weinberger's method in Part 2 to determine the spreading speed without knowing a priori the existence of the corresponding semi-wave solutions. In Part 1 here, we also establish a spreading–vanishing dichotomy.

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1. Introduction and main results

This work consists of two parts, and the current paper is Part 1. The aim of this work is to classify the long-time dynamical behavior to a class of space–time periodic reaction–diffusion equations with free boundaries of the form

$$\begin{cases} u_t = du_{xx} + f(t,x,u), & g(t) < x < h(t), \quad t > 0, \\ u(t,g(t)) = u(t,h(t)) = 0, & t > 0, \\ g'(t) = -\mu u_x(t,g(t)), & t > 0, \\ h'(t) = -\mu u_x(t,h(t)), & t > 0, \\ g(0) = g_0, \quad h(0) = h_0, \quad u(0,x) = u_0(x), & g_0 \le x \le h_0, \end{cases} \tag{1.1}$$

where $x = g(t)$ and $x = h(t)$ are the moving boundaries to be determined together with $u(t,x)$, and μ is a given positive constant. Throughout the paper, the diffusion coefficient d is a positive constant; the reaction term $f : \mathbb{R} \times \mathbb{R} \times \mathbb{R}^+ \mapsto \mathbb{R}$ is continuous, of class $C^{\alpha/2,\alpha}(\mathbb{R} \times \mathbb{R})$ in $(t,x) \in \mathbb{R} \times \mathbb{R}$ locally uniformly in $u \in \mathbb{R}^+$ (with $0 < \alpha < 1$), and of class C^1 in $u \in \mathbb{R}^+$ uniformly in $(t,x) \in \mathbb{R} \times \mathbb{R}$. The basic assumptions on f are:

$$f(t,x,0) = 0 \quad \text{for all } t \in \mathbb{R},\ x \in \mathbb{R}, \tag{1.2}$$

there exists $K > 0$ such that

$$f(t,x,u) \le Ku \quad \text{for all } u \ge 0 \text{ and all } (t,x) \in \mathbb{R}^2. \tag{1.3}$$

Later in the paper, we will assume additionally that there is some constant $M > 0$ such that

$$f(t,x,u) \le 0 \ \text{ for all } t \in \mathbb{R},\ x \in \mathbb{R},\ u \ge M, \tag{1.4}$$

and f is ω-periodic in t and L-periodic in x for some positive constants ω and L, that is,

$$\begin{cases} f(t+\omega,x,u) = f(t,x,u) \\ f(t,x+L,u) = f(t,x,u) \end{cases} \text{for all } (t,x) \in \mathbb{R}^2,\ u \ge 0. \tag{1.5}$$

Let us note that since f is C^1 in u, (1.3) is satisfied whenever (1.2) and (1.4) hold.

The initial function u_0 belongs to $\mathcal{H}(g_0,h_0)$ for some $g_0 < h_0$, where

$$\mathcal{H}(g_0,h_0) := \Big\{\phi \in C([g_0,h_0]) : \phi(g_0) = \phi(h_0) = 0,\ \phi(x) > 0 \text{ in } (g_0,h_0)\Big\}.$$

Free boundary problems of the type (1.1) arise naturally in many applied areas, such as melting of ice in contact with water and spreading of invasive species; see, for example, [4,7,13,29]. In this work, we regard (1.1) as describing the spreading of a new or invasive species over a