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Asymptotic behavior of radially symmetric solutions for a quasilinear hyperbolic fluid model in higher dimensions

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Abstract

We consider the large time behavior of the radially symmetric solution to the equation for a quasilinear hyperbolic model in the exterior domain of a ball in general space dimensions. In the previous paper [2], we proved the asymptotic stability of the stationary wave of the Burgers equations in the same exterior domain when the solution is also radially symmetric. On the other hand, in the 1D-case, a similar asymptotic structure as above to the damped wave equation with a convection term has been established by Ueda [10] and Ueda–Kawashima [11]. Assuming a certain condition on the boundary data on the ball and the behavior at infinity of the fluid, we shall prove that the stationary wave of our quasilinear hyperbolic model is asymptotically stable. The weighted L^2-energy method plays a crucial role in removing such a restriction on the sub-characteristic condition on the stationary wave.

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Keywords: Stationary wave; Asymptotic behavior; Damped wave equation; Galerkin method

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1. Introduction

In this paper, we consider the asymptotic stability of the stationary wave for the equation to the quasi-linear hyperbolic fluid model in general space dimensions under the radially symmetric condition. The motion of the fluid is governed by the transport law

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = \mathrm{div}(2S), \tag{1.1}$$

where $u = (u_1(t,x), \cdots, u_n(t,x))$ denotes the velocity of the fluid at $x = (x_1, \cdots, x_n) \in \Omega$, $t > 0$. Here Ω is a domain in $\mathbb{R}^n$. The tensor S determines the type of the fluid such as incompressible or compressible one together with some parameter describing the physical quantities. For instance, taking $S = \frac{\mu}{2}(\nabla u + {}^t(\nabla u))$ with the viscosity constant $\mu > 0$, we obtain the well-known Burgers equation

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = \mu \Delta u. \tag{1.2}$$

The asymptotic structure to (1.2) has been fully investigated by the first author [2] under the radially symmetric condition when $\Omega = \mathbb{R}^n \setminus B_{r_0}$ for all $n \geq 1$ and all $r_0 > 0$. Indeed, it is shown in [2] that under the condition $u|_{r=r_0} = v_- < v_+ = u|_{r=\infty} = 0$ with $r = |x|$ for the prescribed boundary conditions v_- and v_+, there exists a unique stationary solution $U(r)$ of (1.2) which is asymptotically stable in the sense that every perturbed fluid $u(t,x) = u(t,r)$ of $U(r)$ exists globally in time even for the large initial disturbance at $t = 0$, and satisfies

$$\lim_{t \to \infty} \sup_{r > r_0} |u(t,r) - U(r)| = 0. \tag{1.3}$$

In this paper, we consider a similar asymptotic state for the Cattaneo type law, i.e. S is the solution of the equation

$$\frac{\partial S}{\partial t} + S = \frac{\mu}{2}(\nabla u + {}^t(\nabla u)). \tag{1.4}$$

Then the transport equation (1.1) is reduced to the following Navier–Stokes equation of hyperbolic type:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u - \mu \Delta u = -\left(\frac{\partial^2 u}{\partial t^2} + (u_t \cdot \nabla)u + (u \cdot \nabla)u_t \right) \quad (t > 0, x \in \Omega). \tag{1.5}$$

The systematic theory including (1.5) has been established by Racke–Saal [7] and [8]. Indeed, under the Cauchy problem, they had treated the fully system of the hyperbolic Navier–Stokes equation with its principal part like (1.5) having another non-local unknown pressure function, and proved local well-posedness for large data in [7], and global well-posedness for small data in [8].