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Bifurcation analysis of a spruce budworm model with diffusion and physiological structures [☆]

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Abstract

In this paper, the dynamics of a spruce budworm model with diffusion and physiological structures are investigated. The stability of steady state and the existence of Hopf bifurcation near positive steady state are investigated by analyzing the distribution of eigenvalues. The properties of Hopf bifurcation are determined by the normal form theory and center manifold reduction for partial functional differential equations. And global existence of periodic solutions is established by using the global Hopf bifurcation result of Wu. Finally, some numerical simulations are carried out to illustrate the analytical results.

Keywords: Reaction–diffusion; Delay; Stage structure; Stability; Global Hopf bifurcation

1. Introduction

The spruce budworm is one of the most destructive insects in North American forests, where spruce and balsam fir trees grow. Normally, the spruce budworm exists in low numbers in these forests, kept in check by the predators, primarily birds. However, outbreak of these insects occurs periodically (every 30–40 years lasting for about 10 years) causing billions of dollars loss to for-

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est industry [1–6]. Understanding the dynamics of spruce budworm population is very important to control the growth of budworm and protect spruce and fir forests.

Notice that, as far as the budworm dynamics are concerned, the forest variables may be treated as constants. Also since the birds do not feed exclusively on budworms, their numbers are for the most part independent of the budworm population. In 1979, Ludwig et al. [7] proposed the diffusing budworm population dynamics governed by the equation:

$$u_t = d\Delta u + ru\left(1 - \frac{u}{K}\right) - \frac{Bu^2}{A^2 + u^2}, \tag{1}$$

and investigated the dynamics of Eq. (1) in spatial one dimension. In 2013, Wang and Yeh [8] studied the steady-state problem of (1) and obtained the S-shaped bifurcation diagrams. For some other results about budworm population dynamics we refer to the papers and monographs [9–13].

In fact, the logistic model of budworm population with Holling type III predation function is widely accepted in the literature because of the existence of a stable periodic orbit. But both predation and carrying capacity are unlikely to be a primary cause of budworm population oscillation, see [5]. In 2008, Vaidya and Wu [14] derived a delay differential equation for the matured budworm population from a structured population model and by considering the inactive stage from egg to the second instar caterpillars (L_2) as the immature stage

$$\dot{u} = -Du(t) - \frac{\beta u^2(t)}{\gamma^2 + u^2(t)} + q_1 e^{-\tilde{d}\tau} u(t-\tau) e^{-\alpha_1 u(t-\tau)}. \tag{2}$$

They showed that the simulation results of Eq. (2) are in very good agreement with the real data from the Green River area of New Brunswick, Canada and discussed the role of the parameters on controlling budworm population. They also pointed out that spatial nonhomogeneity should be incorporated as additional factors in further research. So, in this paper, we consider the following diffusive budworm model with Neumann boundary conditions

$$\frac{\partial u(x,t)}{\partial t} = d_1 \Delta u(x,t) - Du(x,t) - \frac{\beta u^2(x,t)}{\gamma^2 + u^2(x,t)} + q_1 e^{-\tilde{d}\tau} u(x, t-\tau) e^{-\alpha_1 u(x,t-\tau)}, \tag{3}$$

where $u(x,t)$ is the mature budworm density at location x and time t, $d_1 > 0$ is the diffusion coefficient, $D > 0$ is the average mortality rate of the mature budworms, $\beta > 0$ represents the predation rate of the birds, $\gamma > 0$ is the budworm population when the predation rate is at half of the maximum, τ is the maturation time delay, $\tilde{d} > 0$ is the average mortality rate of the immature budworms and $b(u) = q_1 u e^{-\alpha_1 u}$ is the birth function of budworms with $q_1, \alpha_1 > 0$.

We would like to mention that, when the stability and Hopf bifurcation at the positive steady state are considered, the difficulty resides in the presence of the dependent delay and the fact that some coefficients in the equations depend upon this delay. Consequently, the characteristic equation of the linearized system has delay-dependent coefficients. As mentioned by Beretta and Kuang [15], models with delay-dependent coefficients often exhibit very rich dynamics as compared to those with constant coefficients. In the analysis, we need to study a series of first degree transcendental polynomial with delay coefficients. The problem of determining the distribution of roots to such polynomials is very complex and there are very few studies on this topic (see [16–21], and references therein).

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