ARTICLE IN PRESS YJDEQ:8655

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Regular dependence of the Peierls barriers on perturbations

Qinbo Chen *, Chong-Qing Cheng

Department of Mathematics, Nanjing University, Nanjing, Jiangsu, 210093, China

Received 20 July 2016; revised 16 December 2016

Abstract

Let f be an exact area-preserving monotone twist diffeomorphism of the infinite cylinder and $P_{\omega,f}(\xi)$ be the associated Peierls barrier. In this paper, we give the Hölder regularity of $P_{\omega,f}(\xi)$ with respect to the parameter f. In fact, we prove that if the rotation symbol $\omega \in (\mathbb{R}\setminus\mathbb{Q})\bigcup(\mathbb{Q}+)\bigcup(\mathbb{Q}-)$, then $P_{\omega,f}(\xi)$ is $1/3$-Hölder continuous in f, i.e.

$$|P_{\omega,f'}(\xi) - P_{\omega,f}(\xi)| \le C\|f' - f\|_{C^1}^{1/3}, \quad \forall \xi \in \mathbb{R}$$

where C is a constant. Similar results also hold for the Lagrangians with one and a half degrees of freedom. As application, we give an open and dense result about the breakup of invariant circles.

MSC: 37Jxx; 70Hxx

Keywords: Peierls barrier; Twist diffeomorphism; Tonelli Lagrangian; Aubry–Mather theory; Invariant circles

1. Introduction

The Peierls barrier $P_{\omega,f}(\xi)$ for the monotone twist diffeomorphism f is a function which can be thought of as a dislocation energy. It measures to which extent the stationary configuration

* Corresponding author.
E-mail addresses: qinboChen1990@gmail.com (Q. Chen), chengcq@nju.edu.cn (C.-Q. Cheng).

http://dx.doi.org/10.1016/j.jde.2016.12.018

$(x_i)_{i\in\mathbb{Z}}$ of rotation symbol ω, subject to the constraint $x_0 = \xi$, is not minimal. In [11], Mather established the modulus of continuity for $P_{\omega,f}$ with respect to the parameter ω, and by applying this property, he gave destruction results for invariant circles under arbitrary small perturbations [12]. For further research, we need more information and properties about the Peierls barriers. In this paper, we prove the Hölder continuity of Peierls barriers with respect to the parameter f, which generalize J. Mather's results in [11,12]. This paper is organized as follows: In Section 2 and Section 3, we introduce the definition of Peierls barrier and some basic properties in Aubry–Mather theory. Our main results are Theorem 4.1 and Theorem 4.9 in Section 4. In Section 5, we give an open and dense result as an application example.

For convenience, we denote by ϑ (mod 1) the standard coordinate of $\mathbb{S}^1 = \mathbb{R}/\mathbb{Z}$ and x the corresponding coordinate of its universal cover $\mathbb{R}$. We will let (ϑ, y) denote the standard coordinates of $\mathbb{S}^1 \times \mathbb{R}$ and (x, y) the corresponding coordinates of the universal cover $\mathbb{R} \times \mathbb{R}$. The dynamical properties of exact area-preserving monotone twist diffeomorphisms of an infinite cylinder $\mathbb{S}^1 \times \mathbb{R}$ have been studied by Mather [9–13] and by Bangert [2]. In the following, we refer to these papers for the definitions and results that we'll need.

1.1. Monotone twist diffeomorphism

Definition 1.1. We call f an exact area-preserving monotone twist diffeomorphism if $f : \mathbb{S}^1 \times \mathbb{R} \longrightarrow \mathbb{S}^1 \times \mathbb{R}$,

$$(\vartheta, y) \longmapsto (\vartheta', y')$$

is a diffeomorphism satisfying the following conditions:

(1) $f \in C^1(\mathbb{S}^1 \times \mathbb{R})$.
(2) The 1-form $y'd\vartheta' - yd\vartheta$ on $\mathbb{S}^1 \times \mathbb{R}$ is exact.
(3) (positive monotone twist) $\frac{\partial \vartheta'(\vartheta,y)}{\partial y} > 0$, for all (ϑ, y).
(4) f twists the cylinder infinitely at either hand. To express this condition, we consider a lift $\bar{f}$ of f to the universal cover $\mathbb{R} \times \mathbb{R}$, $\bar{f}(x, y) = (x', y')$, the condition means that for fixed x,

$$x' \to +\infty \text{ as } y \to +\infty \text{ and } x' \to -\infty \text{ as } y \to -\infty.$$

The positive monotone twist condition has its geometrical meaning. Consider a point $P \in \mathbb{S}^1 \times \mathbb{R}$ and denote by $v_P = (0, 1)$ the vertical vector at P. Let $\beta_f(P)$ denote the angle between v_P and $d_P f \cdot v_P$ (count in the clockwise direction). So the positive monotone twist condition means that

$$0 < \beta_f(P) < \pi$$

everywhere (see Fig. 1).

We denote by $\mathcal{J}$ the class of exact area-preserving monotone twist diffeomorphisms. Let $\mathcal{J}_\beta = \{f \in \mathcal{J} : \beta_f(P) \geq \beta, \text{ for all } P \in \mathbb{S}^1 \times \mathbb{R}\}$. Although $\bigcup_{\beta>0} \mathcal{J}_\beta \subsetneqq \mathcal{J}$, most of our results can be generalized to $\mathcal{J}$ without any difficulty. This is because our main results concern what happens in a compact region K of $\mathbb{S}^1 \times \mathbb{R}$. Thus, for all $f \in \mathcal{J}$, there exists $\beta > 0$ and $g \in \mathcal{J}_\beta$ such that $f|_K = g|_K$.

Please cite this article in press as: Q. Chen, C.-Q. Cheng, Regular dependence of the Peierls barriers on perturbations, J. Differential Equations (2016), http://dx.doi.org/10.1016/j.jde.2016.12.018