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Minimal wave speed for a class of non-cooperative reaction–diffusion systems of three equations

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Abstract

In this paper, we study the traveling wave solutions and minimal wave speed for a class of non-cooperative reaction–diffusion systems consisting of three equations. Based on the eigenvalues, a pair of upper–lower solutions connecting only the invasion-free equilibrium are constructed and the Schauder's fixed-point theorem is applied to show the existence of traveling semi-fronts for an auxiliary system. Then the existence of traveling semi-fronts of original system is obtained by limit arguments. The traveling semi-fronts are proved to connect another equilibrium if natural birth and death rates are not considered and to be persistent if these rates are incorporated. Then non-existence of bounded traveling semi-fronts is obtained by two-sided Laplace transform. Then the above results are applied to some disease-transmission models and a predator–prey model.

Keywords: Three equations; Minimal wave speed; Persistence theory; LaSalle's invariance principle; Applications

1. Introduction

In this paper, we will study the following reaction–diffusion system:

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$$\begin{cases} \frac{\partial}{\partial t}u_1 = d_1\Delta u_1 + f(u_1) - g_1(u), \\ \frac{\partial}{\partial t}u_2 = d_2\Delta u_2 + g_2(u) - \delta_2 u_2, \\ \frac{\partial}{\partial t}u_3 = d_3\Delta u_3 + g_3(u) - \delta_3 u_3, \end{cases} \tag{1.1}$$

where

$$x \in \mathbb{R}^n, \quad u_i = u_i(x,t), \quad i = 1,2,3, \quad u = (u_1,u_2,u_3), \quad \Delta = \sum_{j=1}^{n} \frac{\partial^2}{\partial x_j^2},$$

and all the parameters are positive. The unknowns $u_i, i = 1,2,3$ denote the densities of population, virus, etc. We first present some basic biological assumptions on functions $f(u_1)$ and $g_i(u), i = 1,2,3$. Some notations are first needed to give these assumptions. Define

$$\mathbb{R}^3_+ := \{(u_1,u_2,u_3) : u_1 > 0, u_2 > 0, u_3 > 0\},$$

$$g_{i,j}(u) := \frac{\partial g_i(u)}{\partial u_j}, \quad g_{i,jk}(u) := \frac{\partial^2 g_i(u)}{\partial u_j \partial u_k}, \quad i,j,k = 1,2,3.$$

Then the first three assumptions are as follows.

(A1) $f(\cdot) \in C^1([0,\infty))$, $g_i(\cdot) \in C^2(cl(\mathbb{R}^3_+)), i = 1,2,3$, where $cl(\mathbb{R}^3_+)$ denotes the closure of $\mathbb{R}^3_+$.

(A2) **(I)** Either $f(u_1) \equiv 0$ or $f(u_1)$ satisfies

$$f(K) = 0, \quad f'(K) < 0, \quad f(u_1)(u_1 - K) < 0 \text{ for } u_1 \in (0,K) \cup (K,+\infty),$$

and $f'(0) > 0$ if $f(0) = 0$.

(II) $g_i(u) > 0$, $g_i(u_1,0,0) = 0$ and $g_1(0,u_2,u_3) = 0$ for $u \in \mathbb{R}^3_+, i = 1,2,3$.

(A3) **(I)** $g_{i,j}(u) \geq 0, g_{2,3}(u_1,0,0) > 0, g_{3,2}(u_1,0,0) > 0, i,j = 1,2,3$ for $u \in \mathbb{R}^3_+$.

(II) $g_{i,j1}(u) \geq 0, i,j = 2,3, g_{2,21}(u) + g_{2,31}(u) > 0, u \in \mathbb{R}^3_+$. The Hessian matrices $\mathcal{H}[g_i(u_1,\cdot)](u), i = 1,2,3$ are negative semi-definite for $u \in \mathbb{R}^3_+$, where

$$\mathcal{H}[g_i(u_1,\cdot)](u) := \begin{bmatrix} g_{i,22}(u) & g_{i,23}(u) \\ g_{i,32}(u) & g_{i,33}(u) \end{bmatrix}.$$

Remark 1.1. From (A1) it follows that $g_{i,jk}(u), i,j,k = 2,3$ are bounded in a small neighborhood $\mathscr{U}$ of the equilibrium $E_0(K,0,0)$. In this paper we always suppose (A1)–(A3) are satisfied.

It follows from (A2) and (A3) that the interaction between u_2 and u_3 is cooperative and that both of u_2 and u_3 have negative or non-positive effects on u_1. System (1.1) is consequently non-cooperative or non-monotonic. If $f(u_1) = d(K - u_1)$ or $f(u_1) \equiv 0$, system (1.1) can serve as an SIR model with two progression stages [23, model (16)], an SEIR model [25] or a virus

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