ARTICLE IN PRESS
YJDEQ:8657

ELSEVIER

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Time-periodic boundary layer solutions to singularly perturbed parabolic problems

O.E. Omel'chenko [a,*], L. Recke [b], V.F. Butuzov [c], N.N. Nefedov [c]

[a] *Weierstrass Institute for Applied Analysis and Stochastics, Mohrenstrasse 39, 10117 Berlin, Germany*

[b] *Institute of Mathematics, Humboldt University of Berlin, Rudower Chausse 25, 12489 Berlin, Germany*

[c] *Moscow State University, Faculty of Physics, Department of Mathematics, Vorob'jovy Gory, 19899 Moscow, Russia*

Received 10 August 2016; revised 5 December 2016

Abstract

In this paper, we present a result of implicit function theorem type, which was designed for applications to singularly perturbed problems. This result is based on fixed point iterations for contractive mappings, in particular, no monotonicity or sign preservation properties are needed. Then we apply our abstract result to time-periodic boundary layer solutions (which are allowed to be non-monotone with respect to the space variable) in semilinear parabolic problems with two independent singular perturbation parameters. We prove existence and local uniqueness of those solutions, and estimate their distance to certain approximate solutions.

MSC: 35B25; 35B10; 35K20; 35K58

Keywords: Monotone and non-monotone boundary layers; Two independent singular perturbation parameters; Periodic-parabolic boundary value problem; Implicit function theorem

* Corresponding author.

E-mail addresses: Oleh.Omelchenko@wias-berlin.de (O.E. Omel'chenko), recke@mathematik.hu-berlin.de (L. Recke), butuzov@phys.msu.ru (V.F. Butuzov), nefedov@phys.msu.ru (N.N. Nefedov).

http://dx.doi.org/10.1016/j.jde.2016.12.020

1. Introduction

Upper and lower solution techniques and corresponding monotone iterations are classical methods to prove existence of time-periodic solutions to nonlinear parabolic boundary value problems (see, e.g. [2,11,12,20,22]). In the context of singularly perturbed problems, this approach has allowed to obtain existence and asymptotic expansions of solutions with monotonous boundary [16] and interior [1,4,5,8,17] layers. However, it turned out to be unsuitable for solutions with more complicated boundary layer structure or interior spikes.

In this paper, we present an alternative approach to singularly perturbed periodic-parabolic boundary value problems which is based on fixed point iterations for contractive mappings, i.e. which is an approach of implicit function theorem type. We apply this approach to time-periodic boundary layer solutions (which are allowed to be non-monotone with respect to the space variable) in problems with two independent singular perturbation parameters. More precisely, we consider semilinear parabolic PDEs of the type

$$\mu\partial_t u(t,x) = \nu^2\partial_x^2 u(t,x) + f(t,x,u(t,x),\mu,\nu), \quad (t,x)\in\mathbb{R}\times(0,1) \tag{1.1}$$

with homogeneous Dirichlet boundary conditions

$$u(t,0)=u(t,1)=0, \quad t\in\mathbb{R} \tag{1.2}$$

and periodicity condition in time

$$u(t+1,x)=u(t,x), \quad (t,x)\in\mathbb{R}\times[0,1]. \tag{1.3}$$

Here $\mu,\nu>0$ are two independent small singular perturbation parameters. The right-hand side $f:\mathbb{R}\times[0,1]\times\mathbb{R}\times[0,1]^2\to\mathbb{R}$ is supposed to be C^3-smooth and 1-periodic with respect to its first argument, i.e. with respect to the time variable t. Moreover, we assume that there exists a continuous 1-periodic function $u^0 : \mathbb{R}\times[0,1]\to\mathbb{R}$ such that

$$f(t,x,u^0(t,x),0,0)=0, \quad \partial_u f(t,x,u^0(t,x),0,0)<0, \quad (t,x)\in\mathbb{R}\times[0,1]. \tag{1.4}$$

Our goal is to describe existence, local uniqueness and asymptotic behavior for $\mu,\nu\to 0$ of families (parametrized by μ and ν) $\hat{u}_{\mu,\nu}$ of boundary layer solutions to (1.1)–(1.3), i.e. such that

$$\lim_{(\mu,\nu)\to(0,0)} \hat{u}_{\mu,\nu}(t,x)=u^0(t,x) \quad \text{for all} \quad (t,x)\in\mathbb{R}\times(0,1). \tag{1.5}$$

Such solutions turn out to exist under the following natural assumption (see Theorem 1.1 and Remark 1.3 below): There exist smooth maps $v^0, w^0 : \mathbb{R}\times[0,\infty)\to\mathbb{R}$ such that

$$\left.\begin{aligned} &\partial_y^2 v^0(t,y)+f(t,0,u^0(t,0)+v^0(t,y),0,0)=0, && (t,y)\in\mathbb{R}\times(0,\infty),\\ &v^0(t,0)+u^0(t,0)=v^0(t,\infty)=0, && t\in\mathbb{R},\\ &v^0(t+1,y)=v^0(t,y), && (t,y)\in\mathbb{R}\times[0,\infty) \end{aligned}\right\} \tag{1.6}$$

and

Please cite this article in press as: O.E. Omel'chenko et al., Time-periodic boundary layer solutions to singularly perturbed parabolic problems, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2016.12.020