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Instability in time-delayed switched systems induced by fast and random switching

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Abstract

In this paper, we consider a switched system comprising finitely or infinitely many subsystems described by linear time-delayed differential equations and a rule that orchestrates the system switching randomly among these subsystems, where the switching times are also randomly chosen. We first construct a counter-intuitive example where even though all the time-delayed subsystems are exponentially stable, the behaviors of the randomly switched system change from stable dynamics to unstable dynamics with a decrease of the dwell time. Then by using the theories of stochastic processes and delay differential equations, we present a general result on when this fast and random switching induced instability should occur and we extend this to the case of nonlinear time-delayed switched systems as well.

Keywords: Switched system; Random and fast switching; Time-delay; Stochastic instability; Dwell time

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1. Introduction

In natural and man-made systems, stochastic or periodic fluctuations due to environmental changes are unavoidable. Including the fluctuation influences in mathematical description of these systems usually invites modeling and exploration of switched systems. Mathematically, a switched system comprises a family of finitely or infinitely many subsystems and a rule that orchestrates the system switching among the subsystems. Switched systems along with switching rules can be largely divided into two types: one usually involves switching rules dependent on the loci of the system state variables [1–7], and the other includes rules crucially dependent on a set of switching time instants deterministically or randomly given [8–24].

In this paper, we discuss the stability problem for the second type of switched systems. In fact, many tools have been invented to solve the stability problem. For example, common quadratic Lyapunov function method [8,9] and switched quadratic Lyapunov function method [25,26] were proposed to deal with the stability problem for switched systems under arbitrary switching. A method of multiple Lyapunov functions [10–12] was developed for switched systems in which a concept of fixed switching duration, so-called dwell time, was introduced. It was shown in [13] that, when subsystems are all stable and the dwell time is sufficiently large, the switched system is exponentially stable for any switching law. Later, with an extension of the concept of dwell time to average dwell time, it has been shown that the switched system is still exponentially stable for sufficiently large average dwell time [14]. Aside from the case where all the subsystems are stable, the stability problem for large dwell time is also discussed for switched systems where subsystems could be either stable or unstable [15].

In addition, parallel works on stability have been done for switched systems when time-delay, an omnipresent phenomenon in real-world systems, is introduced. More concretely, it was shown that large dwell time could still guarantee the stability of systems with time-invariant or time-varying delays when all subsystems are stable [16] or when subsystems are either stable or unstable [17–19]. The requirement for sufficiently large values of average dwell time also could ensure the stability of time-delayed systems [20].

All the stability results mentioned above depend on a prerequisite of large dwell times or large average dwell times. Switched systems with large dwell times usually are regarded as slow switched systems; while, systems with small dwell times are regarded as fast switched systems. Also, all these stability results focus on the switching instants that are not stochastically but deterministically specified. However, as reported in the literature, fast and random switching usually brings some interesting dynamical behaviors contrary to the common knowledge. For example, this kind of switching is able to stabilize switched systems even though all the subsystems are unstable [21–23].

Thus, a question naturally arises: "Can fast and random switching bring dynamical behaviors different from those already reported in slow and deterministic time-delayed switched systems?" In this paper, we will fully address this question. More specifically, we will numerically and analytically show how fast and random switching is able to hamper the stability of time-delayed switched systems in spite that the corresponding slow switched systems with all stable subsystems are exponentially stable. This instability result just oppositely corresponds to the stability result induced by fast and random switching in systems with all unstable subsystems but without time-delay [23]. In real applications, addressing the problems of stability and instability is essential for understanding the mechanisms of dynamical behaviors in real systems. For instance, the coupled neurons in the brain area of hippocamp are observed to approach stable synchronization, which always triggers some mental disorders, such as epilepsy and Parkinson diseases

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