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Continuity and asymptotic behaviors for a shallow water wave model with moderate amplitude

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Abstract

In this paper, we study continuity and persistence for a nonlinear evolution equation describing the free surface of shallow water wave with a moderate amplitude, which was proposed by Constantin and Lannes [7]. By the approach for approximate solutions and well-posedness estimates, we obtain two sequences of solution for Constantin–Lannes equation, which are bounded in the Sobolev space $H^s(\mathbb{R})$ with $s > 3/2$, and the distance between the two sequences is lower-bounded by a positive constant for any time t, but converges to zero at the initial time. This implies that the solution map is not uniformly continuous. Furthermore, the solution map for Constantin–Lannes equation is shown Hölder-continuous in H^r-topology for all $0 \le r < s$ with exponent α depending on s and r. In addition, we also investigate the asymptotic behaviors of the strong solutions to Constantin–Lannes equation at infinity within its lifespan provided the initial data in weighted $L^p_\phi := L^p(\mathbb{R}, \phi^p dx)$ spaces.

MSC: 35G25; 35L05; 35Q50

Keywords: Non-uniform dependence; Hölder continuity; Shallow water wave with moderate amplitude; Asymptotic behaviors

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1. Introduction

In this paper, we study the Cauchy problem for the following shallow water wave model with moderate amplitude

$$\begin{cases} \eta_t + \eta_x + \frac{3}{2}\epsilon\eta\eta_x + \iota\epsilon^2\eta^2\eta_x + \kappa\epsilon^3\eta^3\eta_x + \mu(\alpha\eta_{xxx} + \beta\eta_{xxt}) = \epsilon\mu(\gamma\eta\eta_{xxx} + \delta\eta_x\eta_{xx}), \\ \eta(x,0) = \eta_0(x), \end{cases} \tag{1.1}$$

where $\alpha, \gamma, \delta, \iota, \kappa$ and $\beta < 0$ are parameters, ϵ and μ are two dimensionless parameters, which represent the amplitude and shallowness parameters, respectively. The function $u(x,t)$ stands for the free surface elevation, Eq. (1.1) models the propagation of surface waves of moderate amplitude in shallow water regime [7].

The nonlinear water wave is a fascinating subject in the area of nonlinear mathematical physics [25]. Since the governing equations for water waves are usually nearly intractable, the search for suitably simplified models was initiated at the earliest stages of the hydrodynamics development. Until the early twentieth century, the study of water waves was confined almost exclusively to the linear theory. Since linearization failed to explain some important phenomena, several nonlinear models have been proposed to reveal nonlinear behaviors liking breaking waves and solitary waves [7]. The most prominent example is the Korteweg–de Vries (KdV) equation [23], the only member in the wider family of BBM-type equations, which is integrable and relevant for the phenomenon of soliton manifestation [2], in which, the shallowness parameter $\mu \ll 1$ and the amplitude parameter $\varepsilon = O(\mu)$, the shallow water regime of waves of small amplitude.

Since KdV and BBM equations do not model breaking waves, which means the wave remains bounded but its slope becomes unbounded in finite time, several models were proposed to capture this phenomenon. One of the typical models is the Camassa–Holm (CH) equation,

$$u_t - u_{xxt} + c_0u_x + 3uu_x - 2u_xu_{xx} - uu_{xxx} = 0. \tag{1.2}$$

The CH equation is one of the closest relatives model that possesses the regime of shallow water waves with moderate amplitude $\varepsilon = O(\sqrt{\mu})$ and shallowness parameter $\mu \ll 1$. The CH equation was first implicitly included in a bi-Hamiltonian generalization of KdV equation by Fuchssteiner and Fokas [12], and later derived as a model for unidirectional propagation of shallow water over a flat bottom by Camassa and Holm [4]. Similar to the KdV equation, the CH equation also has a bi-Hamiltonian structure [12,4] and is completely integrable in the sense of Lax pair [4]. The orbital stability of solitary waves and the stability of the peakons ($c_0 = 0$) for the CH equation are studied by Constantin and Strauss [8,9]. The advantage of the CH equation in comparison with the KdV equation lies in the fact that the CH equation has peaked solitons (peakons) and models the peculiar wave breaking phenomena [5,6].

Many results have been obtained in the literature for wave propagations with small amplitude, but it is also interesting and important to look at the waves with large amplitudes. The equation derived by Johnson in [21] could be reduced to the CH equation at a certain depth under the fluid surface. One may derive such a similar equation for the free surface valid for waves with moderate amplitude in the shallow water regime. Since quantities of order $\varepsilon = O(\sqrt{\mu})$ are also ones of order $O(\mu)$ for $\mu \ll 1$, the regime of moderate amplitude captures a wider range of wave profiles. In particular, within this regime one expects to get nonlinear models with water

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