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The periodic principal eigenvalues with applications to the nonlocal dispersal logistic equation

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Abstract

In this paper, we study the nonlocal dispersal equation

$$\begin{cases} u_t = \int_{\mathbb{R}^N} J(x-y)u(y,t)dy - u + \lambda u - a(x,t)u^p & \text{in } \bar{\Omega} \times (0,+\infty), \\ u(x,t) = 0 & \text{in } (\mathbb{R}^N \setminus \bar{\Omega}) \times (0,+\infty), \\ u(x,0) = u_0(x) & \text{in } \bar{\Omega}, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, λ and $p > 1$ are constants. The dispersal kernel J is nonnegative. The function $a \in C(\bar{\Omega} \times \mathbb{R})$ is nonnegative and T-periodic in t, but $a(x,t)$ has temporal or spatial degeneracies ($a(x,t)$ vanishes). We first study the periodic nonlocal eigenvalue problems with parameter and establish the asymptotic behavior of principal eigenvalues when the parameter is large. We find that the spatial degeneracy of $a(x,t)$ always guarantees a principal eigenfunction. Then we consider the dynamical behavior of the equation if $a(x,t)$ has temporal or spatial degeneracies. Our results indicate that only the temporal degeneracy can not cause a change of the dynamical behavior, but the spatial degeneracy always causes fundamental changes, whether or not the temporal degeneracy appears.

MSC: 35B40; 35K57; 92D25

Keywords: Positive periodic solution; Stability; Principal eigenvalue; Degeneracy

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Contents

1. Introduction

In this paper, we consider the periodic nonlocal dispersal equation

$$\begin{cases} u_t = J * u - u + \lambda u - a(x,t)u^p & \text{in } \bar{\Omega} \times (0,+\infty), \\ u(x,t) = 0 & \text{in } (\mathbb{R}^N \setminus \bar{\Omega}) \times (0,+\infty), \\ u(x,0) = u_0(x) & \text{in } \bar{\Omega}, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, $p > 1$ and λ is a real parameter, the coefficient $a(x,t)$ is T-periodic in t. The function J is nonnegative and

$$Du(x,t) = J * u(x,t) - u(x,t) = \int_{\mathbb{R}^N} J(x-y)u(y,t)dy - u(x,t)$$

represents a nonlocal dispersal operator. Throughout this paper, we make the following assumptions on J, a and u_0.

$(H1)$ $J \in C(\mathbb{R}^N)$ is nonnegative, $J(0) > 0$, $J(x) = J(-x)$ in $\mathbb{R}^N$ and $\int_{\mathbb{R}^N} J(x)dx = 1$.
$(H2)$ $a \in C(\bar{\Omega} \times [0,\infty))$ is nonnegative, $a(x,t) \not\equiv 0$ and $a(x,t) = a(x,t+T)$ in $\bar{\Omega} \times [0,\infty)$ for some $T > 0$.
$(H3)$ $u_0 \in C(\bar{\Omega})$ is nonnegative and nontrivial.

The nonlocal dispersal equation (1.1) arises typically in population dynamics [2,36]. Let $u(y,t)$ be the density of population at location y at time t, and $J(x-y)$ be the probability distribution of the population jumping from y to x, then $\int_{\mathbb{R}^N} J(x-y)u(y,t)dy$ denotes

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