ARTICLE IN PRESS
YJDEQ:8749

Available online at www.sciencedirect.com

ScienceDirect

J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

A free boundary optimization problem for the ∞-Laplacian

Rafayel Teymurazyan, José Miguel Urbano *

CMUC, Department of Mathematics, University of Coimbra, 3001-501 Coimbra, Portugal

Received 22 September 2016; revised 3 March 2017

Abstract

We study a free boundary optimization problem in heat conduction, ruled by the infinity–Laplace operator, with lower temperature bound and a volume constraint. We obtain existence and regularity results and derive geometric properties for the solution and the free boundaries.
© 2017 Elsevier Inc. All rights reserved.

MSC: 35R35; 35J20; 35J92; 35B65

Keywords: Free boundary problems; Heat conduction; Infinity–Laplace operator; Optimal regularity

1. Introduction

The goal of this paper is to establish existence, and derive geometric properties, for an optimization problem in heat conduction. The problem may be described in the following way: given a non-negative function φ (the temperature profile), our goal is to keep the temperature in a room above φ, by using heating sources inside the room and insulation material of a certain volume outside the room, in a way that minimizes the energy. We consider the energy driven by the infinity–Laplace operator

$$\Delta_\infty u := \sum_{i,j=1}^{n} u_{x_i} u_{x_j} u_{x_i x_j},$$

* Corresponding author.
E-mail addresses: rafayel@mat.uc.pt (R. Teymurazyan), jmurb@mat.uc.pt (J.M. Urbano).

http://dx.doi.org/10.1016/j.jde.2017.03.010
0022-0396/© 2017 Elsevier Inc. All rights reserved.

Please cite this article in press as: R. Teymurazyan, J.M. Urbano, A free boundary optimization problem for the ∞-Laplacian, J. Differential Equations (2017), http://dx.doi.org/10.1016/j.jde.2017.03.010

which, despite being too degenerate to realistically represent a physical diffusion process, has been previously used in the context of free boundary problems, for example in [5], where a dead-core problem is considered. We stress that one of the major difficulties when dealing with Δ_∞ relies precisely on the fact that it diffuses only in the direction of the gradient, which changes point-by-point and depends on the solution itself. It is striking that genuine physical insights can be used to investigate an apparently nonphysical problem with significant mathematical interest.

In mathematical terms, given a smooth bounded domain $\Omega \subset \mathbb{R}^n$, a smooth non-negative function $\varphi : \mathbb{R}^n \to \mathbb{R}$, compactly supported in Ω, and a positive number $\gamma > 0$, we look for a function $u : \mathbb{R}^n \to \mathbb{R}$ that minimizes

$$\mathrm{Lip}(u) \text{ in } \mathbb{K}_\infty, \tag{P_∞}$$

where

$$\mathbb{K}_\infty = \left\{ u \in W^{1,\infty}(\mathbb{R}^n) \,|\, u \geq \varphi,\ |\{u > 0\} \setminus \Omega| \leq \gamma \right\},$$

and such that

$$\begin{cases} \Delta_\infty u = 0 & \text{in} \quad \{u > 0\} \setminus \Omega & \text{(insulation)}, \\ \Delta_\infty u \leq 0 & \text{in} \quad \Omega & \text{(interior heating)}. \end{cases}$$

Here, $\mathrm{Lip}(u)$ is the Lipschitz constant of u

$$\mathrm{Lip}(u) := \sup_{x,y} \frac{|u(x) - u(y)|}{|x - y|},$$

$|E|$ is the n-dimensional Lebesgue measure of the set E, and the relations on $\Delta_\infty u$ are understood in the viscosity sense, according to the next definition.

Definition 1.1. A continuous function u is called a viscosity super-solution (resp. sub-solution) of $\Delta_\infty u = 0$ if for every C^2 function ϕ such that $u - \phi$ has a local minimum at the point x_0, with $\phi(x_0) = u(x_0)$, we have

$$\Delta_\infty \phi(x_0) \leq 0. \quad \text{(resp. } \geq \text{)}$$

A function u is called a viscosity solution if it is both a viscosity super-solution and a viscosity sub-solution.

The problem arises in the study of best insulation devices but motivations also come from plasma physics or flame propagation, for example. The study of optimal configuration problems started decades ago (see [1–3]) and has been developed in recent years to treat optimal design problems ruled by a large class of divergence type operators (see [6,12,15,16]). The case where, instead of the infinity–Laplacian, one has the standard Laplace operator was studied in [20]. Since the fractional Laplacian can be represented as a “Dirichlet to Neumann” map, the techniques used to treat optimal design problems for divergence type operators can be adapted to solve the problem for the fractional Laplacian as well (see [17]). What makes the case of the infinity–Laplacian different is that it does not have a divergence structure, therefore standard tools used in the above mentioned references do not apply, and a different approach is required.

Download English Version:

https://daneshyari.com/en/article/5774229

Download Persian Version:

https://daneshyari.com/article/5774229

Daneshyari.com