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Stability of plane Couette flow for the compressible Navier–Stokes equations with Navier-slip boundary [☆]

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Abstract

This paper is devoted to the stability analysis of the plane Couette flow for the 3D compressible Navier–Stokes equations with Navier-slip boundary condition at the bottom boundary. It is shown that the plane Couette flow is asymptotically stable for small perturbation provided that the slip length, Reynolds and Mach numbers satisfy $\frac{3(1+\widetilde{\nu})\alpha}{\gamma^2(\nu+\alpha)\gamma_0} \leq 1$ and $\frac{2\alpha}{\nu(\nu+\alpha)} \leq 1$ for some constant $\gamma_0 > 0$. In particular, the Reynolds number ν^{-1} can be large if the slip length α is suitably small. This means that the constraint required in [11] on the Reynolds number to guarantee the stability of the plane Couette flow can be relaxed and improved so long as the slip effect at the boundary is involved.

MSC: 76N15; 35Q30

Keywords: Compressible Navier–Stokes equation; Stability; Plane Couette flow; Navier-slip boundary

1. Introduction

This paper is concerned with the existence and asymptotic stability of the barotropic compressible Navier–Stokes equations

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$$\begin{cases} \partial_t \rho + div(\rho v) = 0, \\ \rho \partial_t v + \rho v \cdot \nabla v + \nabla P(\rho) = \nu \Delta v + (\nu + \bar{\nu}) \nabla div v, \end{cases} \tag{1.1}$$

in a three-dimensional infinite layer $\Omega = \mathbb{R}^2 \times (0, 1)$, where we denote the density and velocity by $\rho = \rho(x, t)$ and $v = (v_1(x, t), v_2(x, t), v_3(x, t))^{\perp}$ respectively with $\cdot^{\perp}$ standing for the transposition. Assume that the pressure $P(\rho)$ is a smooth function of ρ satisfying

$$P'(\rho_*) > 0,$$

for a given constant $\rho_* > 0$, ν and $\bar{\nu}$ are the viscosity coefficients satisfying

$$\nu > 0, \quad \frac{2}{3}\nu + \bar{\nu} \geq 0.$$

The corresponding Reynolds number Re, the second Reynolds number $\overline{Re}$ and the Mach number M_a are given by

$$Re = \nu^{-1}, \quad \overline{Re} = \bar{\nu}^{-1}, \quad M_a = \frac{1}{\sqrt{P'(1)}}.$$

We are interested in the stability of the plane Couette flow for compressible Navier–Stokes equations (1.1) with the Navier-slip boundary condition imposed on the bottom boundary, and expect that the boundary effect may play an important role in analyzing the global existence and asymptotical behaviors of solutions near the plane Couette flow. To this end, we assume for simplicity that the flow is driven by the top plate moving along x_1-direction with constant speed $\mathbf{v}_0 = (1, 0, 0)^{\perp}$ and that the boundary $\Sigma \cup \Sigma_b$ is not permeable, namely,

$$v \cdot \mathbf{n} = 0, \quad \text{on } \Sigma \cup \Sigma_b, \tag{1.2}$$

where $\mathbf{n}$ is the outward unit vector normal to the boundary, $\Sigma =: \{x_3 = 1\}$ denotes the top boundary of Ω, and $\Sigma_b =: \{x_3 = 0\}$ the bottom boundary. Moreover, we set the non-slip boundary condition at the top boundary

$$v = \mathbf{v}_0, \quad \text{on } \Sigma; \tag{1.3}$$

and the Navier-slip boundary condition at the bottom boundary

$$S\mathbf{n} \cdot \boldsymbol{\tau} + \alpha v \cdot \boldsymbol{\tau} = 0, \quad \text{on } \Sigma_b, \tag{1.4}$$

where $S = 2\nu \mathbb{D}(v) + (\bar{\nu} div v - P) I_3$ is the stress tensor, I_3 is an identity matrix of order 3, $\mathbb{D}(v)$ is the velocity deformation tensor with elements $D_{ij} = \frac{1}{2}(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$, $\boldsymbol{\tau}$ is any tangent vector orthogonal to $\mathbf{n}$, and $\alpha > 0$ is a constant of slip length or friction coefficient. It should be mentioned that the conditions (1.2) and (1.4) are proposed by Navier [16] and imply that the component of the fluid velocity tangent to the surface is proportional to the rate of strain on the surface. Some recent experiments, generally with typical dimensions microns or smaller, have demonstrated that the phenomenon of slip actually occurs (refer to [4,6] and the references therein).