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Singular radial entire solutions and weak solutions with prescribed singular set for a biharmonic equation ☆

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Abstract

Positive singular radial entire solutions of a biharmonic equation with subcritical exponent are obtained via the entire radial solutions of the equation with supercritical exponent and the Kelvin's transformation. The expansions of such singular radial solutions at the singular point 0 are presented. Using these singular radial entire solutions, we construct solutions with a prescribed singular set for the Navier boundary value problem

$$\Delta^2 u = u^p \quad \text{in } \Omega, \qquad u = \Delta u = 0 \quad \text{on } \partial\Omega$$

where Ω is a smooth open set of $\mathbb{R}^n$ with $n \geq 5$.

MSC: 35J25; 35J20; 35B33; 35B40

Keywords: Radial singular entire solutions; Biharmonic equations; Solutions with prescribed isolated singularities

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1. Introduction

In this paper we study the existence, uniqueness, asymptotic behavior and further qualitative properties of singular radial solutions of the biharmonic equation

$$\begin{cases} \Delta^2 u = u^p & \text{in } \mathbb{R}^n \backslash \{0\} \\ u > 0, & \text{and } \lim_{|x|\to 0} u(x) = +\infty \end{cases} \tag{1.1}$$

where $n \geq 5$ and $\frac{n}{n-4} < p < \frac{n+4}{n-4}$. Moreover, we will construct positive weak solutions with a prescribed singular set for the Navier boundary value problem:

$$\Delta^2 u = u^p \ \text{ in } \Omega, \quad u = \Delta u = 0 \ \text{ on } \partial\Omega \tag{1.2}$$

where Ω is a smooth open set in $\mathbb{R}^n$ with $n \geq 5$. $u \in L^p(\Omega)$ is called a weak solution of (1.2) if the equality

$$\int_\Omega u \Delta^2 \varphi dx = \int_\Omega u^p \varphi dx,$$

holds for any $\varphi \in C^4(\Omega) \cap C^3(\overline{\Omega})$ and $\varphi = \Delta\varphi = 0$ on $\partial\Omega$. In the following, a subset $S \subseteq \Omega$ is called a singular set for a weak solution u of (1.2) if for any $x \in S$, u is not bounded in any neighborhood of x. By the definition and the classical regularity theory, S is then a closed subset of Ω.

We recall that the corresponding second order equation (when $n \geq 3$ and $\frac{n}{n-2} < p < \frac{n+2}{n-2}$)

$$\begin{cases} -\Delta u = u^p & \text{in } \mathbb{R}^n \backslash \{0\} \\ u > 0, & \text{and } \lim_{|x|\to 0} u(x) = +\infty \end{cases} \tag{1.3}$$

is studied in [6] (see e.g. [15], Proposition 1). Among other results, the following classification result is established:

Proposition 1.1. *([6]) Suppose that $\frac{n}{n-2} < p < \frac{n+2}{n-2}$ and u is a solution of (1.3). Then either $u(x) \equiv c_p |x|^{-\frac{2}{p-1}}$ where $c_p = \left[\frac{2}{p-1}\left(n - 2 - \frac{2}{p-1}\right)\right]^{\frac{1}{p-1}}$ or there exists a constant $\beta > 0$ such that*

$$\lim_{|x|\to+\infty} |x|^{n-2} u(x) = \beta. \tag{1.4}$$

Conversely, for any $\beta > 0$, there exists a unique solution $u(x)$ of (1.3) satisfying (1.4).

Using this proposition, Chen and Lin [6] constructed positive weak solutions with a prescribed singular set of the Dirichlet problem

$$\Delta u + u^p = 0 \ \text{ in } \Omega, \quad u = 0 \ \text{ on } \partial\Omega \tag{1.5}$$