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# Well-posedness and decay for the dissipative system modeling electro-hydrodynamics in negative Besov spaces

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**Abstract**

In Guo and Wang (2012) [10], Y. Guo and Y. Wang developed a general new energy method for proving the optimal time decay rates of the solutions to dissipative equations. In this paper, we generalize this method in the framework of homogeneous Besov spaces. Moreover, we apply this method to a model arising from electro-hydrodynamics, which is a strongly coupled system of the Navier–Stokes equations and the Poisson–Nernst–Planck equations through charge transport and external forcing terms. We show that some weighted negative Besov norms of solutions are preserved along time evolution, and obtain the optimal time decay rates of the higher-order spatial derivatives of solutions by the Fourier splitting approach and the interpolation techniques.


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*Keywords:* Navier–Stokes equations; Poisson–Nernst–Planck equations; Electro-hydrodynamics; Well-posedness; Decay; Besov space

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## 1. Introduction

In [10], Y. Guo and Y. Wang developed a new energy approach to establish the optimal time decay rates of the solutions to the Cauchy problem of the heat equation:

$$\begin{cases} \partial_t u - \Delta u = 0, & x \in \mathbb{R}^3,\ t > 0, \\ u(x,0) = u_0(x), & x \in \mathbb{R}^3. \end{cases} \tag{1.1}$$

They proved the following result:

**Theorem 1.1.** *If $u_0 \in H^N(\mathbb{R}^3) \cap \dot{H}^{-s}(\mathbb{R}^3)$ with $N \geq 0$ be an integer and $s \geq 0$ be a real number, then for any real number $\ell \in [-s, N]$, there exists a constant $C_0$ such that*

$$\|\nabla^\ell u(t)\|_{L^2} \leq C_0(1+t)^{-\frac{\ell+s}{2}}. \tag{1.2}$$

Here $H^s(\mathbb{R}^3)$ and $\dot{H}^s(\mathbb{R}^3)$ denote the nonhomogeneous Sobolev space and the homogeneous Sobolev space, respectively.

In this paper, we generalize this new energy approach in the framework of Besov spaces. In order to illustrate this approach, we revisit the heat equation (1.1).

**Theorem 1.2.** *Let $N \geq 0$ be an integer and $s \geq 0$ be a real number, $1 \leq p < \infty$. If $u_0 \in \dot{B}^N_{p,1}(\mathbb{R}^3) \cap \dot{B}^{-s}_{p,1}(\mathbb{R}^3)$, then for any real number $\ell \in [-s, N]$, there exists a constant $C_0$ such that*

$$\|u(t)\|_{\dot{B}^\ell_{p,1}} \leq C_0(1+t)^{-\frac{\ell+s}{2}}. \tag{1.3}$$

**Proof.** Let $\ell \in [-s, N]$. Applying the dyadic operator $\Delta_j$ to the heat equation (1.1), we see that

$$\partial_t \Delta_j u - \Delta\Delta_j u = 0,$$

which taking the standard $L^2$ inner product with $|\Delta_j u|^{p-2}\Delta_j u$ leads to

$$\frac{1}{p}\frac{d}{dt}\|\Delta_j u\|^p_{L^p} - \int_{\mathbb{R}^3} \Delta\Delta_j u|\Delta_j u|^{p-2}\Delta_j u dx = 0.$$

Thanks to [5], there exists a constant $\kappa$ such that

$$-\int_{\mathbb{R}^3} \Delta\Delta_j u|\Delta_j u|^{p-2}\Delta_j u dx \geq \kappa 2^{2j}\|\Delta_j u\|^p_{L^p}.$$

Thus, we obtain

$$\frac{d}{dt}\|\Delta_j u\|_{L^p} + \kappa 2^{2j}\|\Delta_j u\|_{L^p} \leq 0.$$

Multiplying the above inequality by $2^{j\ell}$, then taking $l^1$ norm to the resultant yields that

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