



L-Kuramoto–Sivashinsky SPDEs vs. time-fractional SPIDEs: Exact continuity and gradient moduli, $1/2$ -derivative criticality, and laws

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Abstract

We establish exact, dimension-dependent, spatio-temporal, uniform and local moduli of continuity for (1) the fourth order L-Kuramoto–Sivashinsky (L-KS) SPDEs and for (2) the time-fractional stochastic partial integro-differential equations (SPIDEs), driven by the space–time white noise in one-to-three dimensional spaces. Both classes were introduced—with Brownian-time-type kernel formulations—by Allouba in a series of articles starting in 2006, where he presented class (2) in its rigorous stochastic integral equations form. He proved existence, uniqueness, and sharp spatio-temporal Hölder regularity for the above two classes of equations in $d = 1, 2, 3$. We show that both classes are $(1/2)^-$ Hölder continuously differentiable in space when $d = 1$, and we give the exact uniform and local moduli of continuity for the gradient in both cases. This is unprecedented for SPDEs driven by the space–time white noise. Our results on exact moduli show that the half-derivative SPIDE is a critical case. It signals the onset of rougher modulus regularity in space than both time-fractional SPIDEs with time-derivatives of order $< 1/2$ and L-KS SPDEs. This is despite the fact that they all have identical spatial Hölder regularity, as shown earlier by Allouba. Moreover, we show that the temporal laws governing (1) and (2) are fundamentally different. We relate L-KS SPDEs to the Houdré–Villa bifractional Brownian motion, yielding a Chung-type law of the iterated logarithm for these SPDEs. We use the underlying explicit kernels and spectral/harmonic analysis to prove our results. On one hand, this work builds on the recent works on delicate sample path properties of Gaussian random fields. On the other hand, it builds on and complements Allouba’s earlier works on (1) and (2). Similar regularity results hold for the Allen–Cahn nonlinear members of (1) and (2) on compacts via change of measure.

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Contents

1.	Introduction, statement of results, and preliminaries	1553
1.1.	Two sides of the Brownian-time coin	1553
1.2.	Five questions	1555
1.3.	Main results: answering the questions	1557
1.3.1.	Exact moduli of continuity of L-KS SPDEs and their gradient, and the bifractional Brownian motion link	1558
1.3.2.	Exact moduli of continuity for the time-fractional SPIDEs and their gradient, and their temporal fractional laws	1561
1.4.	The strong local nondeterminism property and modulus of continuity	1566
1.5.	Rigorous kernel stochastic integral equations formulations	1568
2.	Kernels Fourier transforms	1571
3.	The L-KS SPDEs: proofs of Theorems 1.1–1.3	1574
3.1.	Temporal modulus	1575
3.2.	The bifractional Brownian motion link: the case $\vartheta = 0$	1580
3.3.	Spatial modulus	1581
3.3.1.	The case $d = 1$: gradient spatial Hölder and modulus of continuity	1582
3.3.2.	The fractal cases $d = 2, 3$	1584
3.4.	The L-KS gradient temporal Hölder and modulus of continuity	1585
4.	The time-fractional SPIDEs: proofs of Theorems 1.4–1.7	1587
4.1.	Temporal modulus	1588
4.2.	Time-fractional SPIDEs are <i>not</i> bifractional Brownian motions	1591
4.3.	Spatial modulus	1593
4.3.1.	Spectral asymptotics for $0 < \beta < 1/2$	1594
4.3.2.	Spectral asymptotics for the critical fraction $\beta = 1/2$	1595
4.3.3.	Finishing the proofs of Theorems 1.5 and 1.6	1596
4.4.	The time-fractional SPIDE gradient temporal Hölder and modulus of continuity	1599
5.	From linear to time-fractional Allen–Cahn and Swift–Hohenberg equations via measure change	1601
6.	Concluding remarks	1606
	Acknowledgments	1607
	Appendix A. Glossary of frequently used acronyms and notations	1608
	References	1608

1. Introduction, statement of results, and preliminaries

1.1. Two sides of the Brownian-time coin

We delve into delicate regularity properties of paths of fourth order pattern formation stochastic PDEs (SPDEs) and time-fractional slow diffusion stochastic partial integro-differential equa-

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