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# L-Kuramoto–Sivashinsky SPDEs vs. time-fractional SPIDEs: Exact continuity and gradient moduli, 1/2-derivative criticality, and laws

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**Abstract**

We establish exact, dimension-dependent, spatio-temporal, uniform and local moduli of continuity for (1) the fourth order L-Kuramoto–Sivashinsky (L-KS) SPDEs and for (2) the time-fractional stochastic partial integro-differential equations (SPIDEs), driven by the space–time white noise in one-to-three dimensional spaces. Both classes were introduced—with Brownian-time-type kernel formulations—by Allouba in a series of articles starting in 2006, where he presented class (2) in its rigorous stochastic integral equations form. He proved existence, uniqueness, and sharp spatio-temporal Hölder regularity for the above two classes of equations in $d = 1, 2, 3$. We show that both classes are $(1/2)^{-}$ Hölder continuously differentiable in space when $d = 1$, and we give the exact uniform and local moduli of continuity for the gradient in both cases. This is unprecedented for SPDEs driven by the space–time white noise. Our results on exact moduli show that the half-derivative SPIDE is a critical case. It signals the onset of rougher modulus regularity in space than both time-fractional SPIDEs with time-derivatives of order $< 1/2$ and L-KS SPDEs. This is despite the fact that they all have identical spatial Hölder regularity, as shown earlier by Allouba. Moreover, we show that the temporal laws governing (1) and (2) are fundamentally different. We relate L-KS SPDEs to the Houdré-Villa bifractional Brownian motion, yielding a Chung-type law of the iterated logarithm for these SPDEs. We use the underlying explicit kernels and spectral/harmonic analysis to prove our results. On one hand, this work builds on the recent works on delicate sample path properties of Gaussian random fields. On the other hand, it builds on and complements Allouba's earlier works on (1) and (2). Similar regularity results hold for the Allen–Cahn nonlinear members of (1) and (2) on compacts via change of measure.

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**Contents**



## 1. Introduction, statement of results, and preliminaries

### *1.1. Two sides of the Brownian-time coin*

We delve into delicate regularity properties of paths of fourth order pattern formation stochastic PDEs (SPDEs) and time-fractional slow diffusion stochastic partial integro-differential equa-