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Topological features of flows with the reparametrized gluing orbit property

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Abstract

The notions of shadowing, specification and gluing orbit property differ substantially for discrete and continuous time dynamical systems. In the present paper we continue the study of the topological and ergodic properties of continuous flows with the (reparametrized) periodic and nonperiodic gluing orbit properties initiated in [3]. We prove these flows satisfy a weak mixing condition with respect to balls and, if the flow is Komuro expansive, the topological entropy is a lower bound for the exponential growth rate of periodic orbits. Moreover, we show that periodic measures are dense in the set of all invariant probability measures and that ergodic measures are generic. Furthermore, we prove that irrational rotations and some minimal flows on tori and circle extensions over expanding maps satisfy gluing orbit properties, thus emphasizing the difference of this property with respect the notion of specification.

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1. Introduction

The recent revived interest for specification properties in the last few years indicates that the original concept of specification introduced by Bowen [4] is far from generating an old fashioned mechanism to study the topological and ergodic features of the dynamical system. While the strong specification property fails to extend beyond uniformly hyperbolic diffeomorphisms and flows (cf. [20,21]) many other non-uniform notions have been introduced to reflect non-uniform hyperbolicity (cf. [16,14,25,15]). In the time-continuous setting the property of specification is not satisfied even among uniformly hyperbolic basic sets since these may fail to be topologically mixing. Indeed, any suspension flow obtained as the suspension of an Anosov diffeomorphism with a constant roof function is clearly an Anosov flow, hence it is expansive and satisfies the shadowing property, but it misses to be topologically mixing and therefore to satisfy the specification property. Since the specification property has proved to be a very useful tool to study multifractal formalism, thermodynamical formalism and large deviations it is important to create mechanisms that enable us to study these properties in the setting of flows with some weak forms of hyperbolicity.

Motivated by the common features of uniformly hyperbolic flows, in [3] the first and third authors introduced a concept of 'gluing orbit property' which is a topological invariant and much weaker than specification. Among the mechanisms to construct continuous flows with the gluing orbit property we mention: (i) suspension flows of homeomorphisms with the gluing orbit property or specification; and (ii) continuous flows with dense set of periodic orbits and satisfying the shadowing property (see [3,1] for more details). In this way it is possible to provide vast classes of examples of continuous flows with this property, which includes e.g. a C^0-generic subset (hence dense) of Lipschitz vector fields [1].

It is well known that uniformly hyperbolic flows, hence strongly chaotic, have a rich structure on their simplex of invariant probability measures and their thermodynamic formalism is well established [5,18]. The study of the ergodic properties of non-uniformly hyperbolic flows presents key difficulties either in the reduction to the analysis of the discrete-time dynamics of Poincaré return maps, which creates discontinuities for the discrete time dynamics, or by the presence of singularities. For that reason, an extension of such results for wider classes of non-uniformly hyperbolic flows is still a challenge. Some recent contributions in this direction include the fact that periodic measures are dense in the space of invariant probability measures for geodesic flows on non-positively curved manifolds ([7]) and that flows and Hölder continuous potentials for which obstructions to expansiveness and specification have small topological pressure have unique equilibrium states ([6]).

In this paper our purpose is to study some topological and ergodic features of continuous flows with the gluing orbit property and to establish their similitude and differences with respect to flows with specification and also with discrete time dynamical systems. Our first main result is that these systems satisfy some positive lower frequency of visits to balls and this frequency can be taken proportional to the radius of the balls. This condition resembles a weak mixing condition on balls, although the gluing orbit property does not imply on weak mixing (cf. Example 3.1). Moreover, despite the fact that the gluing orbit property need not imply on positive topological entropy (cf. Example 3.2), if the entropy is positive then it implies the number of periodic orbits to grow exponentially. Finally, if the flow is Komuro expansive (a weak notion of expansiveness that allows the presence of singularities) then topological entropy coincides with the exponential growth rate of periodic orbits and it can be computed in any open set. We refer the reader to Theorem A for the precise statements. From the ergodic theory viewpoint, the space of invariant