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Global strong solution to compressible Navier–Stokes equations with density dependent viscosity and temperature dependent heat conductivity

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Abstract

We obtain existence and uniqueness of global strong solution to one-dimensional compressible Navier–Stokes equations for ideal polytropic gas flow, with density dependent viscosity and temperature dependent heat conductivity under stress-free and thermally insulated boundary conditions. Here we assume viscosity coefficient $\mu(\rho) = 1 + \rho^\alpha$ and heat conductivity coefficient $\kappa(\theta) = \theta^\beta$ for all $\alpha \in [0, \infty)$ and $\beta \in (0, +\infty)$.

MSC: 35L65; 35Q30; 76N10

Keywords: Compressible Navier–Stokes equations; Density dependent viscosity; Temperature dependent heat conductivity; Stress-free; Thermally insulated

1. Introduction

In this paper, we investigate the global strong solution to the following one-dimensional compressible Navier–Stokes equations in the Lagrangian coordinates:

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$$\begin{cases} v_t - u_x = 0, \; x \in (0,1), \; t > 0, \\ u_t + p_x = \left(\frac{\mu}{v} u_x\right)_x, \\ (e + \frac{1}{2}u^2)_t + (pu)_x = \left[\frac{\kappa\theta_x + \mu u u_x}{v}\right]_x, \end{cases} \tag{1.1}$$

with stress-free and thermally insulated boundary conditions

$$\left(\frac{\mu}{v} u_x - p\right)(d,t) = 0, \;\; \theta_x(d,t) = 0, \;\; d = 0, 1, \; t \geq 0, \tag{1.2}$$

under prescribed initial conditions

$$(v, u, \theta)(x, 0) = (v_0, u_0, \theta_0)(x), \;\; x \in (0,1). \tag{1.3}$$

Here v, u, θ stand for specific volume, velocity and absolute temperature respectively. The pressure p, the internal energy e, the viscosity coefficient $\mu > 0$ and heat conductivity $\kappa > 0$ are functions of v and θ. The thermodynamic variables v, p, e, s and θ are related by the Gibbs equation $de = ds - pdv$, where s is the specific entropy.

The boundary conditions (1.2) describe the expansion of a finite mass of gas into vacuum. One also considers other kind of boundary conditions

$$u(d,t) = 0, \;\; \theta_x(d,t) = 0, \;\; d = 0, 1, \; t \geq 0, \tag{1.4}$$

and these conditions (1.4) mean that the gas is confined into a fixed tube with impermeable gas.

For constant coefficients, Kazhykhove and Shelukhin established global existence and uniqueness of smooth solutions for arbitrarily large and smooth data in the seminal work [1] under boundary conditions (1.4). Later on, this result was generalized to cases when coefficients may depend on the Lagrangian space variable x by Amosov and Zlotnik [2,3].

However, based on the celebrated Chapman–Enskog expansion for the first order approximation, the viscosity μ and heat conductivity κ are functions of temperature, cf. [4,5]. Particularly, if the inter-molecule potential is proportional to r^{-a} with r being the molecule distance, then

$$\mu, \; \kappa \sim \theta^{\frac{a+4}{2a}}. \tag{1.5}$$

Note that for Maxwellian molecules ($a = 4$) the dependence is linear, while for elastic spheres ($a \to +\infty$) the dependence is like $\sqrt{\theta}$.

The above dependence brings great difficulty and big challenge to mathematicians, especially temperature dependence on μ. One attempts to work out this problem under the assumption that the viscosity μ depends only on density first.

Dafermos [6] and Dafermos and Hsiao [7] considered the density dependence on μ, κ and temperature dependence on κ where they asked whether κ is bounded as well as uniformly bounded away from zero. Later on, Kawohl [8], Jiang [9,10] and Wang [11] established the global existence of smooth solutions for (1.1), (1.3) with boundary condition of either (1.2) or (1.4) under the assumption $\mu(v) \geq \mu_0 > 0$ for any $v > 0$ and κ may depend on both density and temperature. If, however, $\mu(v)$ tends to zero as $v \to \infty$, the situation becomes more delicate. For the case $\mu(\rho) = \rho^\alpha$, Jiang [12] showed that if $\mu(v)$ does not decrease to 0 too rapidly, then the smooth solution still exists globally in time when $0 < \alpha < \frac{1}{4}$. Later on, Qin and Yao [13,14] enlarged the range of α to $(0, \frac{1}{2})$. However, all these works [8–14] assumed there are constants