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Strong solutions and instability for the fitness gradient system in evolutionary games between two populations

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Abstract

In this paper, we study a fitness gradient system for two populations interacting via a symmetric game. The population dynamics are governed by a conservation law, with a spatial migration flux determined by the fitness. By applying the Galerkin method, we establish the existence, regularity and uniqueness of global solutions to an approximate system, which retains most of the interesting mathematical properties of the original fitness gradient system. Furthermore, we show that a Turing instability occurs for equilibrium states of the fitness gradient system, and its approximations.

MSC: 92D25; 35D35; 35K20

Keywords: Two interacting populations; Strong solutions; Galerkin approximation; Turing instability

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1. Introduction

The ability of living things to move spatially during their struggle to survive is an inherent aspect of most biological systems, with implicit connections to evolution [8]. The fact that multiple species are moving simultaneously, some in pursuit of others, brings an additional richness to ecological dynamics. The particular mechanisms of this motion manifest themselves at the population level as dispersal or migration relations, written as a spatial flux which depends on various effects, including heterogeneous environmental conditions, spatial distribution of resources, and mutually attractive or repulsive interactions between individuals, among many other considerations [2,8]. The challenge for mathematical modeling is to realistically capture the relevant aspects of these effects, while nonetheless producing a set of equations which are both tractable and provide insight into the phenomena [10].

Partial differential equations have been developed to model populations interacting in a spatially extended region. Among such models, one of the first (called SKT model for short) determined by species fitness appeared in Shigesada et al. [29], who studied a Lotka–Volterra system of interacting species in a homogeneous environment. For the SKT model, Lou and Ni [14,15] showed the existence and nonexistence of nonconstant steady states, and obtained the limit of nonconstant steady states. The global existence of smooth solutions was proved by Kim [11] and Shim [26] in one dimension, Lou et al. [16] in two dimensions, and Lou and Winkler [18] in three dimensions. When the environment itself is spatially inhomogeneous, the case of one species moving up a resource gradient while the other disperses randomly was modeled by Cantrell et al. [4] based on an earlier single equation approach by Belgacem and Cosner [1]; Kareiva and Odell proposed a cross-diffusion model for predator–prey interaction [13]. We also refer to [3,10,19,21,22,28,20] and references therein.

Evolutionary game theory provides a specific form of the fitness for each population, based on the payoff matrix of the game which defines their mutual interactions [30]. Consider a population of individuals who are playing a game in competition. Every individual has a choice of m possible pure strategies available, and at each instant every individual is using one of these strategies. For each strategy i, p_i denotes the proportion of individuals who are, at that moment, using strategy i. In a symmetric evolutionary game, the fitness of strategy i is the expected payoff for an individual playing strategy i, written as f_i, where the payoff matrix is defined by

$$\mathbf{A} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mm} \end{pmatrix}.$$

We adopt the fitness function as defined by Taylor & Jonker [32] and Vickers [34], where the fitness for an individual playing strategy i is defined as the expected per capita payoff: $f_i = (\mathbf{Ap})_i$, where $\mathbf{p} = (p_1, \cdots, p_m)$.

In this paper we consider two populations, described by density functions u and v, who choose one from two strategies ($m = 2$). The local fitness for each population defined above is written as

$$f_1(u,v) = \frac{a_{11}u + a_{12}v}{u+v}, \qquad f_2(u,v) = \frac{a_{21}u + a_{22}v}{u+v}. \tag{1.1}$$

We assume that

$$a_{11} - a_{12} > 0, \qquad a_{21} - a_{22} > 0, \tag{1.2}$$

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