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Stability of periodic steady-state solutions to a non-isentropic Euler–Poisson system

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Abstract

We study the stability of periodic smooth solutions near non-constant steady-states for a non-isentropic Euler–Poisson system without temperature damping term. The system arises in the theory of semiconductors for which the doping profile is a given smooth function. In this stability problem, there are no special restrictions on the size of the doping profile, but only on the size of the perturbation. We prove that small perturbations of periodic steady-states are exponentially stable for large time. For this purpose, we introduce new variables and choose a non-diagonal symmetrizer of the full Euler equations to recover dissipation estimates. This also allows to make the proof of the stability result very simple and concise.

MSC: 35B40; 35J05; 35Q35

Keywords: Non-isentropic Euler–Poisson system; Global smooth solution; Stability; Large steady-state solution; Energy estimate

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1. Introduction

In this paper, we study a stability problem for a non-isentropic Euler–Poisson system arising in the modeling of plasmas and semiconductors consisting of electrons. The symbols n, u and ϕ stand for the density, the velocity and the electric potential of the electrons, and p, e, θ, the thermodynamic variables, stand for the pressure, the internal energy and the absolute temperature, respectively. The total energy E is defined by

$$E = \frac{1}{2} n|u|^2 + ne.$$

We consider the problem in a periodic domain $\mathbb{T}^d = (\mathbb{R}/\mathbb{Z})^d$. The system satisfied by these variables reads (see [3,9])

$$\begin{cases} \partial_t n + \operatorname{div}(nu) = 0, \\ \partial_t (nu) + \operatorname{div}(nu \otimes u) + \nabla p = n\nabla\phi - nu, \\ \partial_t E + \operatorname{div}(Eu + pu) = nu \cdot \nabla\phi - (E - ne_L), \\ -\Delta\phi = b(x) - n, \end{cases} \tag{1.1}$$

for $t > 0$ and $x \in \mathbb{T}^d$, where $e_L > 0$ is a constant and $b(x)$ is the doping profile for semiconductors. We assume b is sufficiently smooth and satisfies $b \geq \text{const.} > 0$ on $\mathbb{T}^d$. The system is supplemented by the following periodic condition

$$t = 0: \qquad (n, u, \theta) = \big(n_0(x), u_0(x), \theta_0(x)\big), \quad x \in \mathbb{T}^d. \tag{1.2}$$

Problem (1.1)–(1.2) has been studied in [1,17]. For convenience, we consider the case of ideal polytropic gas

$$p = Rn\theta, \qquad e = C_v\theta, \tag{1.3}$$

where $R > 0$ and $C_v > 0$ are physical constants. Introduce

$$\gamma = \frac{R + C_v}{C_v} > 1, \quad \theta_L = \frac{e_L}{C_v}.$$

By the state equation (1.3), for smooth solutions in any non-vacuum field, the momentum and energy equations in (1.1) can be written as

$$\partial_t u + u \cdot \nabla u + \frac{1}{n}\nabla p = \nabla\phi - u, \tag{1.4}$$

$$\partial_t \theta + u \cdot \nabla\theta + (\gamma - 1)\theta \operatorname{div} u = \frac{\gamma - 1}{2R}|u|^2 - (\theta - \theta_L). \tag{1.5}$$

The last terms $-u$ in (1.4) and $-(\theta - \theta_L)$ in (1.5) imply the velocity dissipation and the temperature dissipation in energy estimates, respectively.

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