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YJDEQ:8696

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J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

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Refuge versus dispersion in the logistic equation

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Received 14 June 2016; revised 23 January 2017

Abstract

In this paper we consider a logistic equation with nonlinear diffusion arising in population dynamics. In this model, there exists a refuge where the species grows following a Malthusian law and, in addition, there exists also a non-linear diffusion representing a repulsive dispersion of the species. We prove existence and uniqueness of positive solution and study the behavior of this solution with respect to the parameter λ, the growth rate of the species. Mainly, we use bifurcation techniques, the sub-supersolution method and a construction of appropriate large solutions.

MSC: 35J60; 35B32; 92B05

Keywords: Non-linear diffusion; Bifurcation; Sub-supersolution method; Large solutions; Population dynamics

1. Introduction

Reaction–diffusion models have been used to study the behavior of a population living in a habitat. Denoting by $\Omega \subset \mathbb{R}^N$, $N \geq 1$, a bounded and regular domain, the habitat and by $u(x)$ the

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1 Bolsista da CAPES – Proc. nº BEX 6377/15-7.

2 The authors have been partially supported for the project MTM2015-69875-P (MINECO/FEDER, UE).

3 The author has been partially supported for the project CNPq-Proc. 400426/2013-7.

http://dx.doi.org/10.1016/j.jde.2017.02.012

density of the individuals of the species at the location $x \in \Omega$, the classical model can be written as follows

$$\begin{cases} -\Delta(\varphi(x,u)) = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where φ and f are regular functions in $\Omega \times \mathbb{R}$. The term on the left side of (1) represents the diffusion of the species, that is, its spatial movement. In the model described by (1), the diffusion depends on the position $x \in \Omega$ and the population density u. The nonlinear diffusion function φ can have several different shapes, depending on the nature of behavioral interactions between organisms (see [1] and [2]). For instance, according to [2], if individuals move completely independently of each other, φ is characterized by a linear function of density u, that is, it increases with u at a constant rate. This case is called *simple* or *linear diffusion*. If interactions between moving individuals are repulsive, then the movement rate will increase with the population density, since at high densities organisms continuously come into contact and induce each other to disperse. In this case the diffusion rate φ_u will increase with the density. Similarly, if the movement is aggregative, the diffusivity will initially decline as u increases.

On the other hand, $f(x,u)$ is the *reaction term* and it represents the local rate of reproduction per individual, in other words, per capita population growth rate.

Specifically, in this paper we analyze the following elliptic equation

$$\begin{cases} -\Delta(u + a(x)u^r) = \lambda u - b(x)u^p & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

where $p, r > 1$, $b \in \mathcal{C}(\overline{\Omega}, \mathbb{R}_+)$ and $a \in \mathcal{C}^2(\overline{\Omega}, \mathbb{R}_+)$ are regular functions that can vanish on some subsets of Ω. In this specific case, $f(x,u) = \lambda u - b(x)u^p$, it is the well-known logistic reaction term and, in the context of population dynamics, λ is the intrinsic rate of natural increase of the species and

$$C(x) \equiv \frac{\lambda}{b(x)}$$

denotes the maximum density supported locally by available resources, that is, the carrying capacity. Thus, the region where $b(x) = 0$ can be understood as a refuge area for the species, i.e., the carrying capacity is infinite. For more details about problems with refuge areas see the pioneering papers [3] and [4], where the problem of the refuge was addressed by the first time, see also [1,5,6] and the recent book [7].

In the nonlinear diffusion term $\varphi(x,u) = u + a(x)u^r$, the function a denotes the type of diffusion movement of the species: linear when $a = 0$ and repulsive when $a > 0$. Thus, the set $\{x \in \Omega;\ a(x) > 0\}$ is a region where the species must avoid agglomeration. In our discussion, we will consider different configurations for the refuge area and the zone with repulsive movement to analyze how these sets affect the persistence of the species.

Let us recall the main known results about (2). For this, given a regular subdomain $D \subset \Omega$, we denote by $\lambda_1[-\Delta; D]$ the principal eigenvalue of the Laplacian in D under homogeneous Dirichlet boundary conditions, $\lambda_1[-\Delta; D] = \infty$ when $D = \emptyset$ and, by simplicity, $\lambda_1 = \lambda_1[-\Delta; \Omega]$.

When $a \equiv 0$ in Ω, denoting $\Omega_{b+} := \{x \in \Omega;\ b(x) > 0\}$ and $\Omega_{b0} := \Omega \setminus \overline{\Omega}_{b+}$, (2) becomes the classical logistic equation with linear diffusion and refuge. For instance, suppose that Ω_{b0} is