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Stability of stationary solution for the compressible viscous magnetohydrodynamic equations with large potential force in bounded domain

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Abstract

In this paper, we consider the 3-D compressible viscous magnetohydrodynamic (MHD) equations with some large potential force in bounded rigid vessel. We firstly construct the non-constant stationary solutions of the compressible viscous MHD equations under suitable constitutive assumptions. Next, a critical energy identity is established to achieve a universal stability criterion of the stationary solution. In this case, the stationary solution is exponential stable for any large external potential force. Finally, we show the well-posedness of the initial boundary value problem for the compressible viscous MHD equations with the large potential force, provided that the prescribed initial data is close to the stationary solution. It implies that the set satisfying the stability criterion is not empty.

Keywords: Magnetohydrodynamic equations; Initial-boundary problem; Potential force; Stationary solution; Stability criterion

Contents

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1. Introduction

In this paper, we are interested in the following compressible viscous MHD equations in domain Ω

$$\begin{cases} \partial_t \rho + \operatorname{div}(\rho \mathbf{u}) = 0, \\ \partial_t(\rho \mathbf{u}) + \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u}) + \nabla P = (\nabla \times \mathbf{H}) \times \mathbf{H} + \mu \Delta \mathbf{u} + (\mu + \lambda)\nabla \operatorname{div} \mathbf{u} + \rho F, \\ \partial_t \mathbf{H} - \nabla \times (\mathbf{u} \times \mathbf{H}) = -\nu \nabla \times (\nabla \times \mathbf{H}), \quad \operatorname{div} \mathbf{H} = 0. \end{cases} \tag{1.1}$$

Here the variable $t \geq 0$ is time, and x is the spatial coordinate. Ω is a three dimensional bounded domain of $\mathbb{R}^3$ with the smooth boundary $\partial\Omega$. $\otimes$ represents the diadic symbol, $\times$ is the cross product. ρ, $\mathbf{u}$ and $\mathbf{H}$ are the density, the velocity and the magnetic field respectively. The pressure $P = P(\rho)$ is a smooth positive function satisfying $P'(\rho) > 0$ for $\rho > 0$. $F(x) = (F_1(x), F_2(x), F_3(x))$ is a given external force. The constants μ and λ are the shear and bulk viscosity coefficients of the flow satisfying the physical restrictions $\mu > 0$ and $2\mu + 3\lambda \geq 0$, while the constant $\nu > 0$ is the magnetic diffusion coefficient. The isentropic compressible viscous MHD equations models the dynamics of compressible quasi-neutrally ionized fluids under the influence of electromagnetic fields. It has a very wide applications in physics, for example, the liquid metals and the cosmic plasmas, see [20,23,28,30].

Due to the physical importance, there are many studies on the multi-dimensional compressible magnetohydrodynamic equations. Kawashima [19] obtained the global existence of smooth solutions to the general electromagnetic fluid equations in the two-dimensional case when the initial data are small perturbations of some given constant state. Umeda et al. [32] obtained the global existence and the time decay of smooth solutions to the three-dimensional compressible linearized MHD equations. Zhang and Zhao [38] established some decay estimates of solutions for the 3-D compressible isentropic MHD equation. Fan et al. [9,6] obtained the local strong solution to the nonlinear compressible MHD equations. Further, Fan and Yu [8] showed the global variational solutions to the compressible MHD equations. Li and Yu [26], and Chen and Tan [1] not only established the global existence of classical solutions, but also obtained the time decay rates for the three-dimensional compressible MHD equations by assuming the initial data belong to L^1 and $L^q (q \in [1, \frac{6}{5}))$ respectively. Tan and Wang [31] improved the time decay rates and showed the optimal time decay rates for the higher-order spatial derivatives of solutions if the initial perturbation belongs to $H^N \cap H^{-s} (N \geqslant 3, s \in [0, \frac{3}{2}))$. Li, Xu and Zhang [24] obtained global classical solutions to 3D compressible MHD equations with large oscillations and vacuum. Hao [13] discussed the well-posedness to the compressible viscous MHD system in Besov space. Cai and Tan [2,3] obtained time periodic solutions to the three-dimensional equations of compressible MHD flows. Xu and Zhang [36] gave a blow-up criterion for 3D compressible magnetohydrodynamic equations with vacuum. The global weak solutions to the nonlinear compressible MHD with general initial data were established by Hu and Wang [16] and Fan and Yu