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On the stability of weakly hyperbolic invariant sets

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Abstract

The dynamical object which we study is a compact invariant set with a suitable hyperbolic structure. Stability of weakly hyperbolic sets was studied by V. A. Pliss and G. R. Sell (see [1,2]). They assumed that the neutral, unstable and stable linear spaces of the corresponding linearized systems satisfy Lipschitz condition. They showed that if a perturbation is small, then the perturbed system has a weakly hyperbolic set K^Y, which is homeomorphic to the weakly hyperbolic set K of the initial system, close to K, and the dynamics on K^Y is close to the dynamics on K. At the same time, it is known that the Lipschitz property is too strong in the sense that the set of systems without this property is generic. Hence, there was a need to introduce new methods of studying stability of weakly hyperbolic sets without Lipschitz condition. These new methods appeared in [16–20]. They were based on the local coordinates introduced in [18] and the continuous on the whole weakly hyperbolic set coordinates introduced in [19]. In this paper we will show that even without Lipschitz condition there exists a continuous mapping h such that $h(K) = K^Y$.

Keywords: Dynamical systems; Hyperbolicity; Invariant sets; Stability; Weakly hyperbolic sets; Small perturbations

Contents

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1. Introduction

Stability problems for weakly hyperbolic invariant sets are quite important for the theory of ordinary differential equations. Many papers about such problems are published during the last forty years (see [1–14]); several ones are classical for this area. In the majority of the above papers, it is assumed that the neutral, unstable and stable linear spaces of the corresponding linear system satisfy the Lipschitz condition. In particular, this refers to [1,2]; the present paper develops the results of those two papers. On the other hand, it is known that such a restriction is not quite essential. It is proved that the set of systems without Lipschitz condition is generic. Hence in any neighborhood of any weakly hyperbolic invariant set of a system that satisfies the Lipschitz condition (see examples in [1]) there is a "similar" weakly hyperbolic invariant set of a system that does not satisfy the Lipschitz condition. This motivates the interest to the non-Lipschitz case.

This direction is originated in [16]: the stability of leaf invariant sets of two-dimensional periodic systems without the Lipschitz property is studied under the assumption that the dimension of the unstable linear space is equal to zero. In the said work we introduce notion of leaves of weakly hyperbolic invariant sets and show that if a C^1-perturbation is sufficiently small, then a neighborhood of a leaf of a weakly hyperbolic invariant set of the unperturbed system contains a leaf of the perturbed system. Moreover, we prove the existence of a map h such that it is continuous on the leaf and $h(\Upsilon) = \Upsilon^Y$, where Υ and Υ^Y are the leaves of the unperturbed and perturbed systems respectively. In [17] it is proved that the set $K^Y = \bigcup_{\Upsilon \in K} \Upsilon^Y$ is closed. Thus, in [16] and [17] it is shown that the leaf invariant set is stable even if the Lipschitz condition is not satisfied.

Further research directions are as follows. First, we investigated whether it is possible, assuming that the Lipschitz condition is not satisfied, to construct a map h such that it is continuous on the set K (apart from each separate leaf). Secondly, we have to extend the results of [16,17] to the n-dimensional case. In [18] the results of [16,17] are extended to three-dimensional periodic systems. In [19] the continuous map $h : K \to K^Y$ is constructed for three-dimensional periodic systems. To do that, we introduce continuous coordinates (apart from the local coordinates used in [18]) by means of Perron disks; it is known from [5] that the said disks form a foliation in a sufficiently small neighborhood of any leaf even if the Lipschitz condition is not satisfied. In [20] we complete the series [16–19], considering the n-dimensional system of ordinary differential equations, possessing a hyperbolic attractor (a hyperbolic attractor is a weakly hyperbolic invariant set such that the dimension of its unstable linear space is equal to zero), and