ARTICLE IN PRESS YJDEQ:8551

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Journal of Differential Equations

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Coexistence and exclusion of stochastic competitive Lotka–Volterra models ☆

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Received 13 April 2015; revised 5 September 2016

Abstract

This work derives sufficient conditions for the coexistence and exclusion of a stochastic competitive Lotka–Volterra model. The conditions obtained are close to necessary. In addition, convergence in distribution of positive solutions of the model is also established. A number of numerical examples are given to illustrate our results.

MSC: 34C12; 60H10; 92D25

Keywords: Ergodicity; Coexistence; Exclusion; Lotka–Volterra; Competition; Stationary distribution

1. Introduction

Cooperation, predator–prey, and competition are three main interactions among species in eco-systems. Among them, competition is one of the most popular interactions. Such interactions occur when two or more species compete for the same resource such as food, shelter, nesting sites, etc. Due to competition, the growth of a species is depressed in the presence of others. Traditionally, competitive interactions are modeled by systems of ordinary differential equations

☆ This research was supported in part by the National Science Foundation under grant DMS-1207667.
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http://dx.doi.org/10.1016/j.jde.2016.10.005

Please cite this article in press as: D.H. Nguyen, G. Yin, Coexistence and exclusion of stochastic competitive Lotka–Volterra models, J. Differential Equations (2016), http://dx.doi.org/10.1016/j.jde.2016.10.005

known as the Lotka–Volterra models. For instance, a competitive Lotka–Volterra model for two species takes the form

$$\begin{cases} dx(t) = x(t)\big(a_1 - b_1 x(t) - c_1 y(t)\big)dt \\ dy(t) = y(t)\big(a_2 - b_2 y(t) - c_2 x(t)\big)dt, \end{cases} \tag{1.1}$$

where $x(t)$ and $y(t)$ represent the densities of the two species at time t, a_1, and $a_2 > 0$ are intrinsic growth rates, and b_1 and $b_2 > 0$ are intra-specific competition rates while c_1 and $c_2 > 0$ represent the inter-specific competition. An important question regarding the competitive interaction is whether the species co-exist or a competitive exclusion occurs. This question has been addressed fully for the deterministic model given by (1.1). We state a result whose proof can be found in [9] or [19].

Proposition 1.1. *Let* $\lambda_1 := a_2 - c_2\dfrac{a_1}{b_1}$ *and* $\lambda_2 := a_1 - c_1\dfrac{a_2}{b_2}$.

(i) *If* $\lambda_1 > 0$ *and* $\lambda_2 > 0$, *all positive solutions* $(x(t), y(t))$ *to* (1.1) *converge to the unique positive equilibrium* $\left(\dfrac{a_1c_2 - a_2b_1}{c_1c_2 - b_1b_2}, \dfrac{a_2c_1 - a_1b_2}{c_1c_2 - b_1b_2}\right)$.

(ii) *If* $\lambda_1 > 0$ *and* $\lambda_2 < 0$, *all positive solutions* $(x(t), y(t))$ *converge to* $(0, \dfrac{a_2}{b_2})$.

(iii) *If* $\lambda_1 < 0$ *and* $\lambda_2 > 0$, *all positive solutions* $(x(t), y(t))$ *converge to* $\left(\dfrac{a_1}{b_1}, 0\right)$.

(iv) *If* $\lambda_1 < 0$ *and* $\lambda_2 < 0$, *there is an unstable manifold (called the separatrix) splitting the interior of the positive quadrant* $\mathbb{R}^{2,\circ}_+$ *into two regions. Solutions above the separatrix converge to* $\left(0, \dfrac{a_2}{b_2}\right)$, *while solutions below the separatrix tend to* $\left(\dfrac{a_1}{b_1}, 0\right)$.

Proposition 1.1 indicates that in case (i), the interspecific competition is not too strong, so the two species coexist. For the rest of the cases, the competitive exclusion takes place. In particular, in case (iv), one population with starting advantage (i.e., its initial density is sufficiently larger than that of the other) will eventually win, while the other will be extinct. In addition, in case (ii) or (iii), one species always dominates the other.

In the past decade, besides deterministic models, stochastic ecology models have gained increasing attention to depict more realistically eco-systems. The main thoughts are that such systems are often subject to environmental noise. Various types of environmental noises have been considered. General Lotka–Volterra models perturbed by white noise have been studied in [7,12,15,17,18], while the authors in [22,27,32,33] go further by considering the effect of both white and colored noises to the Lotka–Volterra models. Assuming that the population may suffer sudden environmental shocks, e.g., earthquakes, hurricanes, epidemics, etc., Bao et al. in [2] consider competitive system with jumps. Meanwhile, Tran and Yin [31] use a Wonham filter to deal with a regime-switching Lotka–Volterra model in which the switching is a hidden Markov chain. In the aforementioned papers, some nice estimates on moment and pathwise asymptotic behaviors have been given. Some efforts have also been devoted to providing conditions for permanence and extinction of the species as well as the existence of stationary distribution. Nevertheless, no conditions as sharp as their deterministic counterpart (cf. Proposition 1.1) have been obtained. Motivated by the needs, this paper aims to provide the classification for a stochastic

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