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Global stability of prey-taxis systems

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Abstract

In this paper, we prove the global boundedness and stability of the predator–prey system with prey-taxis in a two-dimensional bounded domain with Neumann boundary conditions. By deriving an entropy-like equality and a boundedness criterion, we show that the intrinsic interaction between predators and preys is sufficient to prevent the population overcrowding even the prey-taxis is included and strong. Furthermore, by constructing appropriate Lyapunov functionals, we show that prey-only steady state is globally asymptotically stable if the predation is weak, and the co-existence steady state is globally asymptotically stable under some conditions (like the prey-taxis is weak or the prey diffuses fast) if the predation is strong. The convergence rates of solutions to the steady states are derived in the paper.

MSC: 35A01; 35B40; 35B44; 35K57; 35Q92; 92C17

Keywords: Predator–prey system; Prey-taxis; Boundedness; Global stability; Lyapunov functional; Decay rates

1. Introduction

Prey-taxis, the movement of predators towards the area with higher density of prey population, plays important roles in biological control and ecological balance such as regulating prey (pest) population or incipient outbreaks of prey or forming large-scale aggregation for survival, cf. [11,25,31]. It was first observed in the field experiment by Karevia and Odell reported in the

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paper [15] where a PDE prey-taxis model was derived to interpret the heterogeneous aggregative patterns due to the interactions between individual ladybugs (predators) and aphids (prey) subject to the so-called area-restricted search strategy. In order to put the detailed individual field observations into a meaningful and tractable population-level model, Karevia and Odell [15] treated the prey-taxis as biased random walks which can incorporate micro-scale observations of individuals. Then passing to the continuum limit, they derived a PDE model which, augmented with the predator–prey interaction, can be formulated as:

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u\rho(u,v)\nabla v) + G_1(u,v), \\ v_t = D\Delta v + G_2(u,v), \end{cases} \tag{1.1}$$

where $u = u(x,t)$ denotes the predator density at position x and time $t > 0$ and $v = v(x,t)$ the prey population density; the term $-\nabla \cdot (u\rho(u,v)\nabla v)$ stands for the prey-taxis with a coefficient $\rho(u,v)$ which may depend on the predator or prey density and D is the prey diffusion rate. The functions $G_1(u,v)$ and $G_2(u,v)$ describe the population interactions between the predator and the prey.

Ecological/biological population interactions can be defined as either intra-specific or inter-specific. The former occurs between individuals of the same species, while the later between different species. The predator–prey population interaction, including both intra-specific or inter-specific interactions, possesses the following prototypical form

$$G_1(u,v) = \gamma u F(v) - uh(u),\ G_2(u,v) = f(v) - uF(v)$$

where $uF(v)$ represents the inter-specific interaction, $uh(u)$ and $f(v)$ accounts for the intra-specific interaction. Specifically $F(v)$ is the so-called functional response function accounting for the intake rate of predators as a function of prey density, $h(u)$ is the predator mortality rate function and $f(v)$ is the prey growth function; the parameters $\gamma > 0$ denotes the intrinsic predation rate. The most widely used forms of $F(v)$ in the literature are:

$$\begin{aligned} &F(v) = v \text{ (Lotka–Volterra type or Holling type I);} \\ &F(v) = \frac{v}{\lambda + v} \text{ (Holling type II);}\ \ F(v) = \frac{v^m}{\lambda^m + v^m} \text{ (Holling type III)} \end{aligned} \tag{1.2}$$

with constants $\lambda > 0$ and $m > 1$. The predator mortality rate function $h(u)$ is typically of the form

$$h(u) = \theta + \alpha u \tag{1.3}$$

where $\theta > 0$ accounts for the natural death rate and $\alpha \geq 0$ denotes the rate of death resulting from the intra-specific competition (also called density-dependent death, e.g. see [20]). The prey growth function $f(v)$ is usually assumed to be negative for large v due to the limitation of resource (or crowding effect) and typical forms are

$$\begin{aligned} &f(v) = \mu v(1 - v/K) \text{ (Logistic type);} \\ &f(v) = \mu v(1 - v/K)(v/k - 1) \text{ (Bistable or Allee effect type)} \end{aligned} \tag{1.4}$$

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