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Global well-posedness and decay of smooth solutions to the non-isothermal model for compressible nematic liquid crystals

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Abstract

The Cauchy problem for the three-dimensional non-isothermal model for compressible nematic liquid crystals is considered. Existence of global-in-time smooth solutions is established provided that the initial datum is close to a steady state $(\bar{\rho}, \mathbf{0}, \bar{\mathbf{d}}, \bar{\theta})$. By using the L^q–L^p estimates and the Fourier splitting method, if the initial perturbation is small in H^3-norm and bounded in L^q $(q \in [1, \frac{6}{5}))$ norm, we obtain the optimal decay rates for the first and second order spatial derivatives of solutions. In addition, the third and fourth order spatial derivatives of director field $\mathbf{d}$ in L^2-norm are achieved.
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1. Introduction

The nematic liquid crystals flows are regarded as slow moving particles where the fluid velocity and the alignment of the particles affect each other. Ericksen and Leslie have established the

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hydrodynamic theory of liquid crystals in [15,16,32,33]. In this paper, we consider the following three-dimensional non-isothermal model for compressible nematic liquid crystals:

$$\begin{cases} \rho_t + \operatorname{div}(\rho \mathbf{u}) = 0, \\ (\rho \mathbf{u})_t + \operatorname{div}(\rho \mathbf{u} \otimes \mathbf{u}) = \operatorname{div}\mathbb{H}, \\ \mathbf{d}_t + \mathbf{u} \cdot \nabla \mathbf{d} = \tilde{\gamma}(\Delta \mathbf{d} + |\nabla \mathbf{d}|^2 \mathbf{d}), \\ (\rho\theta)_t + \operatorname{div}(\rho\theta \mathbf{u}) + \operatorname{div}\mathbf{q} = \mathbb{H} : \nabla \mathbf{u}, \end{cases} \tag{1}$$

where $\rho \in \mathbb{R}$ is the density function of the fluid, $\mathbf{u} \in \mathbb{R}^3$ is the velocity, $\mathbf{d} \in \mathrm{S}^2$ represents the director field for the averaged macroscopic molecular orientations and θ stands for the absolute temperature. The scalar function $P = R\rho\theta$ is the pressure with the gas constant $R > 0$. The flux $\mathbf{q}$ is given by

$$\mathbf{q} = -\kappa \nabla \theta.$$

$\mathbb{H}$ is the Cauchy stress tensor and defined by

$$\mathbb{H} = \mathbb{S} - \xi(\nabla \mathbf{d} \odot \nabla \mathbf{d} - \frac{1}{2}|\nabla \mathbf{d}|^2 \mathbb{I}) - P\mathbb{I},$$

where $\mathbb{S}$ is the conventional Newtonian viscous stress tensor,

$$\mathbb{S} = \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) + \lambda(\operatorname{div}\mathbf{u})\mathbb{I}.$$

Here, the positive constants ξ, $\tilde{\gamma}$, κ denote the competition between kinetic energy and potential energy, the microscopic elastic relation time for the molecular orientation field, and the ratio of the heat conductivity coefficient over the heat capacity, respectively. For simplicity, we set $\tilde{\gamma} = 1$. $\mathbb{I}$ is the identity matrix, and the constants $\mu > 0$ and λ are the shear viscosity and the second viscosity coefficients satisfying the usual condition $\lambda + \frac{2}{3}\mu \geq 0$. The symbol $\otimes$ denotes the Kronecker multiplication and $(\mathbf{u} \otimes \mathbf{u})_{ij} = u^i u^j$. The denotation $\nabla \mathbf{d} \odot \nabla \mathbf{d}$ accounts for a matrix whose ij-th entry $(1 \leq i, j \leq 3)$ is $\partial_i \mathbf{d} \cdot \partial_j \mathbf{d}$. To complete the system (1), the initial data are given by

$$(\rho, \mathbf{u}, \theta, \mathbf{d})(x, t)|_{t=0} = (\rho_0(x), \mathbf{u}_0(x), \theta_0(x), \mathbf{d}_0(x)). \tag{2}$$

Furthermore, as the spatial variable tends to infinity, we assume

$$\lim_{|x| \to \infty} (\rho_0 - \bar{\rho}, \mathbf{u}_0, \mathbf{d}_0 - \bar{\mathbf{d}}, \theta_0 - \bar{\theta})(x) = 0, \tag{3}$$

where $(\bar{\rho}, \mathbf{0}, \bar{\mathbf{d}}, \bar{\theta})$ is the steady-state solution.

Physically, $|\nabla \mathbf{d}|^2 \mathbf{d}$ is preferred to the penalty term, and $\mathbf{d}$ shall satisfy the constraint $|\mathbf{d}| = 1$. For the compressible case, it's easy to see that $\operatorname{div}(\nabla \mathbf{d} \odot \nabla \mathbf{d} - \frac{1}{2}|\nabla \mathbf{d}|^2 \mathbb{I}) = \Delta \mathbf{d} \cdot (\nabla \mathbf{d})^T$, while for the incompressible case, the term $\frac{1}{2}|\nabla \mathbf{d}|^2 \mathbb{I}$ can be absorbed into the pressure P. Feireisl et al. in [17] proposed a non-isothermal model of nematic liquid crystals and only investigated the global

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