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Limiting behavior of non-autonomous stochastic reaction–diffusion equations on thin domains

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Abstract

This paper deals with the limiting behavior of stochastic reaction–diffusion equations driven by multiplicative noise and deterministic non-autonomous terms defined on thin domains. We first prove the existence, uniqueness and periodicity of pullback tempered random attractors for the equations in an $(n+1)$-dimensional narrow domain, and then establish the upper semicontinuity of these attractors when a family of $(n+1)$-dimensional thin domains collapses onto an n-dimensional domain.

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1. Introduction

Let $\mathcal{Q}$ be a smooth bounded domain in $\mathbb{R}^n$ and $\mathcal{O}_\varepsilon$ be the following $n+1$ dimensional region

$$\mathcal{O}_\varepsilon = \left\{ x = \left(x^*, x_{n+1}\right) | x^* = (x_1, \dots, x_n) \in \mathcal{Q},\ 0 < x_{n+1} < \varepsilon g\left(x^*\right) \right\},$$

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where $g \in C^2(\overline{\mathcal{Q}}, (0, +\infty))$ and $0 < \varepsilon \leq 1$. Since $g \in C^2(\overline{\mathcal{Q}}, (0, +\infty))$, there exist two positive constants γ_1 and γ_2 such that

$$\gamma_1 \leq g\left(x^*\right) \leq \gamma_2, \quad \forall x^* \in \overline{\mathcal{Q}}. \tag{1.1}$$

Denote $\mathcal{O} = \mathcal{Q} \times (0, 1)$ and $\widetilde{\mathcal{O}} = \mathcal{Q} \times (0, \gamma_2)$ which contains $\mathcal{O}_\varepsilon$ for $0 < \varepsilon \leq 1$.

In this paper we study the limit of asymptotical behavior of stochastic reaction–diffusion equation with multiplicative noise on $\mathcal{O}_\varepsilon$ as ε tends to zero:

$$\begin{cases} d\hat{u}^\varepsilon - \Delta\hat{u}^\varepsilon dt = \left(F\left(t, x, \hat{u}^\varepsilon\right) + G\left(t, x\right)\right) dt + \hat{u}^\varepsilon \circ dW, \quad x \in \mathcal{O}_\varepsilon,\ t > \tau, \\ \frac{\partial \hat{u}^\varepsilon}{\partial \nu_\varepsilon} = 0, \quad x \in \partial\mathcal{O}_\varepsilon, \end{cases} \tag{1.2}$$

with initial condition

$$\hat{u}^\varepsilon(\tau, x) = \hat{u}^\varepsilon_\tau(x), \quad x \in \mathcal{O}_\varepsilon, \tag{1.3}$$

where $\tau \in \mathbb{R}$, ν_ε is the unit outward normal vector to $\partial\mathcal{O}_\varepsilon$, F is nonlinear function defined on $\mathbb{R} \times \widetilde{\mathcal{O}} \times \mathbb{R}$, G is a function defined on $\mathbb{R} \times \widetilde{\mathcal{O}}$, W is a two-sided real-valued Wiener process on a probability space. The stochastic equation (1.2) is understood in the sense of Stratonovich integration. The domain $\mathcal{O}_\varepsilon$ is the so-called thin domain when ε is small.

As $\varepsilon \to 0$, the thin domain $\mathcal{O}_\varepsilon$ collapses to an n-dimensional domain and it is natural to ask what happens to the family of continuous cocycle $\hat{\phi}_\varepsilon$ generated by (1.2) and (1.3) as $\varepsilon \to 0$. Does there exist a limiting cocycle ϕ_0 for $\varepsilon = 0$? And if so, how can we determine ϕ_0 and what is the relationship between $\hat{\phi}_\varepsilon$ and ϕ_0?

In this paper, we will address these issues. We will see that the limiting behavior of the equation is determined by the following system on the lower dimensional spatial domain $\mathcal{Q}$:

$$\begin{cases} du^0 - \frac{1}{g}\sum_{i=1}^{n}(gu^0_{y_i})_{y_i} dt = \left(F\left(t, y^*, 0, u^0\right) + G\left(t, y^*, 0\right)\right) dt + u^0 \circ dW, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad y^* = (y_1, \dots, y_n) \in \mathcal{Q},\ t > \tau, \\ \frac{\partial u^0}{\partial \nu_0} = 0, \quad y^* \in \partial\mathcal{Q}, \end{cases} \tag{1.4}$$

with initial condition

$$u^0(\tau, y^*) = u^0_\tau(y^*), \quad y^* \in \mathcal{Q}, \tag{1.5}$$

where ν_0 is the unit outward normal vector to $\partial\mathcal{Q}$. Note that $u^0_{y_i}$ means $\frac{\partial u^0}{\partial y_i}$ in (1.4) and similar notation will be used throughout this paper.

Thin domains problems have been considered by many authors from different points of view, including modeling, control and homogenization of equations. Such problems have been investigated by many different approaches like asymptotic expansions and singular perturbations. The systematic study of the asymptotic behavior of deterministic dissipative systems on thin domains was initiated by Hale and Raugel [18,19]. Later on, their results were extended to various problems, see for instance, [1,3,10,20,23–26].

Stochastic partial differential equations arise naturally in a wide variety of applications where uncertainties or random influences, called noises, are taken into account. The study of global