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Instability of solitary wave solutions for derivative nonlinear Schrödinger equation in endpoint case

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Abstract

We study the stability theory of solitary wave solutions for a type of the derivative nonlinear Schrödinger equation

$$i\partial_t u + \partial_x^2 u + i|u|^2\partial_x u + b|u|^4 u = 0.$$

The equation has a two-parameter family of solitary wave solutions of the form

$$e^{i\omega_0 t + i\frac{\omega_1}{2}(x-\omega_1 t) - \frac{i}{4}\int_{-\infty}^{x-\omega_1 t}|\varphi_\omega(\eta)|^2 d\eta}\varphi_\omega(x-\omega_1 t).$$

The stability theory in the frequency region of $|\omega_1| < 2\sqrt{\omega_0}$ was studied previously. In this paper, we prove the instability of the solitary wave solutions in the endpoint case $\omega_1 = 2\sqrt{\omega_0}$, in which the elliptic equation of φ_ω is "zero mass".
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1. Introduction

In this paper, we study the stability theory of solitary wave solutions for the derivative nonlinear Schrödinger equation:

$$i\partial_t u + \partial_x^2 u + i|u|^2\partial_x u + b|u|^4 u = 0, \qquad t \in \mathbb{R}, x \in \mathbb{R}, \tag{1.1}$$

where $b > 0$. It describes an Alfvén wave and appears in plasma physics, nonlinear optics, and so on (see [16,17]). When $b = 0$, by a suitable gauge transformation, (1.1) is transformed to the standard derivative nonlinear Schrödinger equation:

$$i\partial_t u + \partial_x^2 u + i\partial_x(|u|^2 u) = 0. \tag{1.2}$$

It was proved in [9–11,19] that the Cauchy problem for (1.1) or (1.2) is locally well-posed in the energy space $H^1(\mathbb{R})$. See also [5,22,23,20,21,1] for some of the previous or extended results. Furthermore, it was proved in [25] that (1.2) is globally well-posed in the energy space $H^1(\mathbb{R})$ when the initial data satisfies $\|u_0\|_{L^2} < 2\sqrt{\pi}$. See [3,4,7,8,11,15,19,24] for the related results. See also [13,14] for the stability results on the generalized derivative nonlinear Schrödinger equation.

The solution $u(t)$ of (1.1) satisfies three conservation laws

$$E(u(t)) = E(u_0), P(u(t)) = P(u_0), M(u(t)) = M(u_0)$$

for all $t \in [0, T_{max})$, where

$$\begin{aligned} E(u(t)) &= \frac{1}{2}\|\partial_x u\|_{L^2}^2 - \frac{1}{4}(i|u|^2\partial_x u, u)_{L^2} - \frac{b}{6}\|u\|_{L^6}^6, \\ P(u(t)) &= \frac{1}{2}(i\partial_x u, u)_{L^2}, \\ M(u(t)) &= \frac{1}{2}\|u\|_{L^2}^2. \end{aligned}$$

It is known (see for examples [6,2,25]) that (1.2) has a two-parameter family of solitary wave solutions of the form:

$$\widetilde{u}_\omega(t,x) = e^{i\omega_0 t + i\frac{\omega_1}{2}(x-\omega_1 t) - \frac{3}{4}i\int_{-\infty}^{x-\omega_1 t}|\widetilde{\varphi}_\omega(\eta)|^2 d\eta}\widetilde{\varphi}_\omega(x-\omega_1 t),$$

where $\omega = (\omega_0, \omega_1) \in \Omega := \{(\omega_0, \omega_1) \in \mathbb{R}^+ \times \mathbb{R} : \omega_1^2 \le 4\omega_0\}$, and $\widetilde{\varphi}_\omega$ is the solution of

$$-\partial_x^2\varphi + (\omega_0 - \frac{\omega_1^2}{4})\varphi + \frac{\omega_1}{2}|\varphi|^2\varphi - \frac{3}{16}|\varphi|^4\varphi = 0.$$

In [2], Colin and Ohta proved that $\widetilde{u}_\omega(t,x)$ is stable when $\omega_1^2 < 4\omega_0$. See also [6] for the case when $\omega_1 < 0$ and $\omega_1^2 < 4\omega_0$. The stability theory on the endpoint case $\omega_1^2 = 4\omega_0$ remains open.

When $b > 0$, (1.1) has a two-parameter family of solitary wave solutions of the form:

$$u_\omega(t,x) = e^{i\omega_0 t + i\frac{\omega_1}{2}(x-\omega_1 t) - \frac{i}{4}\int_{-\infty}^{x-\omega_1 t}|\varphi_\omega(\eta)|^2 d\eta}\varphi_\omega(x-\omega_1 t), \tag{1.3}$$

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