ARTICLE IN PRESS YJDEQ:8575

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J. Differential Equations ••• (••••) •••–•••

Journal of Differential Equations

www.elsevier.com/locate/jde

Uniqueness for elliptic problems with locally Lipschitz continuous dependence on the solution

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Received 11 June 2016

Abstract

We consider a class of elliptic equations of the type

$$-\operatorname{div}\big(a(x,u,\nabla u)\big) = f - \operatorname{div} g$$

with Dirichlet boundary conditions and with f belonging to $L^1(\Omega)$. Using the framework of renormalized solutions we prove the uniqueness of a solution when the usual Lipschitz condition on the function a is relaxed.
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MSC: 35J60; 35R05; 35A02

Keywords: Uniqueness result; Elliptic equations; Renormalized solution; Integrable data

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http://dx.doi.org/10.1016/j.jde.2016.10.029
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Please cite this article in press as: F. Feo, O. Guibé, Uniqueness for elliptic problems with locally Lipschitz continuous dependence on the solution, J. Differential Equations (2016), http://dx.doi.org/10.1016/j.jde.2016.10.029

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1. Introduction

Let us consider the following class of nonlinear elliptic problems

$$\begin{cases} -\operatorname{div}\big(a(x,u,\nabla u)\big) = f - \operatorname{div} g & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded open subset of $\mathbb{R}^N$, $p > 1$, $f \in L^1(\Omega)$, $g \in (L^p(\Omega))^N$ and where the function $a(x,s,\xi)$ defines a (generalized) Leray–Lions operator $u \mapsto -\operatorname{div}(a(x,u,\nabla u))$.

It is well known that considering $f \in L^1(\Omega)$ requires a convenient framework of solution. Indeed a solution in the sense of distribution may exist (see [3]), but it is not unique in general (see the counter example in [18]). Here we use the notion of renormalized solution developed by DiPerna–Lions in [11,12] for first order equations and adapted to elliptic equations for L^1 data in [16] and for Radon measure data in [9].

As far as the uniqueness question is concerned and when the right-hand side $f - \operatorname{div} g$ belongs to $W^{-1,p}(\Omega)$ the uniqueness of the weak solution is proved for example in [6,8,4] when $1 < p \leq 2$ and when $a(x,s,\xi)$ is a Lipschitz continuous function with respect to s. Indeed it is well known that the uniqueness of the weak solution to $-\operatorname{div}(a(x,u,\nabla u) + \varphi(u)) = f$ may fail in general when $p > 2$ (see the counter example in [6,4]). However under additional conditions on the operator a, without the presence of $\varphi(u)$ and under a sign condition on the data $f - \operatorname{div} g$ the case $p > 2$ is addressed in [7].

When the right-hand side belongs to L^1 the concept of renormalized solutions allowed to establish uniqueness results for quasi-linear elliptic problem in [2] (see also [17] in the framework of entropy solutions) and for operators with $a(x,s,\xi) = a(x,s)|\xi|^{p-2}\xi$ for $1 < p \leq 2$ in [14]. In all this papers the uniqueness of the solution is obtained under a fairly general but global condition on a. This condition relies on a global differential inequality and allows highly oscillating or/and increasing coefficient $a(x,s)$ with respect to s. Moreover for this specific operators the uniqueness results in [2,17,14] are more general that the ones in the variational case. In [10] a generalization to anisotropic operators is proposed. The authors consider the following equation

$$-\sum_{i=1}^{N} \partial_{x_i}\big(a_i(x,u)|\partial_{x_i}u|^{p_i-2}\partial_{x_i}u\big) = f - \operatorname{div} g$$

with $f \in L^1(\Omega)$, $1 < p_i < \infty$ (there exists $p_{i_0} \leq 2$), $g_i \in L^{p_i'}(\Omega)$ and where $a_i(x,s)$ are Carathéodory functions such that $a_i(x,s) \geq \lambda > 0$ a.e. on $\Omega \times \mathbb{R}$ and verify a very local Lipschitz condition with respect to s for every $1 \leq i \leq N$. Even if the results established in [2,17,14] are very general the main improvement in [10] is the very local condition.

In the present paper we prove the uniqueness of the renormalized solution to (1.1) under some structural conditions on the operator and a very local Lipschitz condition on a with respect to s, when $1 < p \leq 2$. We also deal with the case $p > 2$ under an additional sign condition on the data. Here we relax this global Lipschitz continuity with a local one. We use mainly the techniques developed in [2,14,10]. The main novelty of the present paper with respect to the mentioned references is that we obtain the uniqueness of the renormalized solution to (1.1) when the function a depends also on ∇u and when $a(x,s,\xi)$ is very local Lipschitz continuous with respect to s. Indeed to our knowledge in the above mentioned references the main uniqueness

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