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On maximal parabolic regularity for non-autonomous parabolic operators

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Abstract

We consider linear inhomogeneous non-autonomous parabolic problems associated to sesquilinear forms, with discontinuous dependence of time. We show that for these problems, the property of maximal parabolic regularity can be extrapolated to time integrability exponents $r \neq 2$. This allows us to prove maximal parabolic L^r-regularity for discontinuous non-autonomous second-order divergence form operators in very general geometric settings and to prove existence results for related quasilinear equations.

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Keywords: Non-autonomous evolution equations; Parabolic initial boundary value problems; Maximal parabolic regularity; Extrapolation of maximal parabolic regularity

Contents

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1. Introduction

In this paper we are interested in maximal parabolic regularity for non-autonomous parabolic equations like

$$u'(t) + \mathcal{A}(t)u(t) = f(t),$$

for almost every $t \in (0, T)$, where $u(0) = 0$, $f \in L^r((0, T); X)$ and the operators $\mathcal{A}(t)$ all have the same domain of definition D in a Banach space X. If the operator function $\mathcal{A}(\cdot)$ is constant, these equations may be solved using the concept of maximal parabolic regularity, see for example [19,9,8,55,28,27,68,20,12,23,62]. This theory extends to cases in which the dependence

$$(0, T) = J \ni t \mapsto \mathcal{A}(t) \in \mathcal{L}(D; X) \tag{1}$$

is continuous, see [40, Chapter VI], [61,6] and the survey in [63] and it is a powerful tool for solving corresponding nonlinear equations, see [17,60,7,48,45,47,52,33].

If the continuity of (1) is violated, things are much less understood, the only classical exception being the case that the summability exponent in time, r, equals 2 and that X and D are Hilbert spaces (see Proposition 4.1 below). For results in the Banach space case and recent achievements see [3,42,10,11,25,24,39,37,38] and the references therein. Most of these results are either based on the specific structure of m-th order parabolic PDEs, or they can be seen as perturbation results with respect to an *autonomous* parabolic operator.

The spirit of our paper is perturbative in a different sense: not the operator is changed, but the Banach space. This enables us to extrapolate maximal parabolic regularity for whole classes of non-autonomous operators simultaneously. Remarkably, Gröger proved in [42] that maximal parabolic $L^r(J; W^{-1,q}_{\mathfrak{D}})$-regularity for second-order divergence-form operators is preserved in case of non-smooth, time-dependent coefficients, if one deviates from $q = r = 2$ to temporal and spatial integrability exponents $q = r$ in an interval $[2, 2 + \varepsilon)$ that depends on the ellipticity constant and the L^∞-norm of the coefficient function. Here, we provide an abstract extrapolation strategy that includes general non-autonomous operators corresponding to sesquilinear forms, and we extend the results of Gröger to indices $q \neq r \in (2 - \varepsilon, 2 + \varepsilon)$ and more general geometric settings for mixed boundary conditions.

In [24], Dong considered second-order parabolic equations in divergence form with leading coefficients measurable in time and partially small in the BMO semi-norm in the spatial variables. His results provide unique solutions in $L^r(J; W^{1,p}_0(\Omega))$ for arbitrary $1 < r, p < \infty$, with "reduced" time regularity, where Ω may be the whole space or a Lipschitz domain with