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Global well-posedness and large time behavior of classical solutions to the Vlasov–Fokker–Planck and magnetohydrodynamics equations

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Abstract

We are concerned with the global well-posedness of the fluid-particle system which describes the evolutions of disperse two-phase flows. The system consists of the Vlasov–Fokker–Planck equation for the dispersed phase (particles) coupled to the compressible magnetohydrodynamics equations modelling a dense phase (fluid) through the friction forcing. Global well-posedness of the Cauchy problem is established in perturbation framework, and rates of convergence of solutions toward equilibrium, which are algebraic in the whole space and exponential on torus, are also obtained under some additional conditions on initial data. The existence of global solution and decay rate of the solution are proved based on the classical energy estimates and Fourier multiplier technique, which are considerably complicated and some new ideas and techniques are thus required. Moreover, it is shown that neither shock waves nor vacuum and concentration in the solution are developed in a finite time although there is a complex interaction between particle and fluid.

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Keywords: Magnetohydrodynamics; Vlasov–Fokker–Planck equation; Global well-posedness; Rate of convergence

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1. Introduction

Fluid-particle interaction systems have been proposed to describe the behavior of sprays [4,32], aerosols [1] combustion theory [34] or more generically two phase flows [3,35] where one phase (disperse) can be considered as a suspension of particles onto the other one (dense) thought as a fluid. This kind of systems has been used in a wide range of applications from engineering, medicine, geophysics and astrophysics [2,14] and has attracted numerous mathematical studies in modeling and analysis.

In this paper, we consider a system of partial differential equations describing the motions of uncharged particles in a viscous inhomogeneous compressible conducting fluid. The system consists of the viscous magnetohydrodynamics (MHD) equations coupled with the Vlasov–Fokker–Planck equation with friction force in the following form:

$$n_t + div(nu) = 0, \tag{1.1}$$

$$\begin{aligned}(nu)_t + div(nu \otimes u) - \mu\Delta u - (\kappa+\mu)\nabla div u + \nabla P \\ = (\nabla \times B) \times B + \int_{\mathbb{R}^3} (v-u)F dv,\end{aligned} \tag{1.2}$$

$$B_t - \nabla \times (u \times B) = -\nabla \times (\nu \nabla \times B), \tag{1.3}$$

$$div B = 0, \tag{1.4}$$

$$F_t + v\cdot\nabla F = div_v[(v-u)F + \nabla_v F]. \tag{1.5}$$

Here, the unknowns are (n, u, B, F), where $n = n(t,x) > 0$, $u = u(t,x) \in \mathbb{R}^3$, $B = B(t,x) \in \mathbb{R}^3$, for $t \geq 0, x \in \Omega$ denote the mass density, velocity field of the fluid and the magnetic field respectively, and $F = F(t,x,v) \geq 0$ for $t \geq 0, x \in \Omega, v \in \mathbb{R}^3$ denotes the density distribution function of particles in the phase space. The spatial domain $\Omega = \mathbb{R}^3$ or $\mathbb{T}^3$. $P = P(n) = c_\gamma n^\gamma$ is the material pressure with $\gamma > 1$ and $c_\gamma > 0$, κ and μ are the constant viscosity coefficients, $\mu > 0$, $3\kappa + 2\mu \geq 0$, $\nu > 0$ is the magnetic diffusivity acting as a magnetic diffusion coefficient of the magnetic field.

There exists a rich literature on the study of the fluid–particle flows and MHD equations. For the incompressible fluid–particle model, Hamdache established global existence and large-time behavior of solutions for the Vlasov–Stokes system [18]. The global existence of weak solutions to the incompressible Vlasov–Navier–Stokes system on periodic domain was proved in [6]. This result was extended to bounded domain in [37]. The global existence of classical solutions near equilibrium for the incompressible Navier–Stokes–Vlasov–Fokker–Planck system was established in [17] and the corresponding inviscid case was studied in [7]. For compressible model, Mellet–Vasseur obtained the global existence and asymptotic analysis of weak solutions for compressible Navier–Stokes–Vlasov–Fokker–Planck system in [30,31]. The existence of global classical solutions close to an equilibrium and exponential decay for this model was given in [9]. Duan–Liu studied the global well-posedness of small solution in the perturbation framework for inviscid flow of this system [13]. Recently, Li–Mu–Wang studied the global strong solutions to a general kinetic-fluid model in [26].

For MHD equations, in the one-dimensional case, the compressible MHD equations have been studied in many papers [10,15,33]. Kawashima [22] obtained the global existence of smooth solutions to the general electromagnetic fluid equations in the two-dimensional case when the initial