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# Steady flow for shear thickening fluids in domains with unbounded sections

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**Abstract**

We solve the stationary Stokes and Navier–Stokes equations for non-Newtonian incompressible fluids with shear dependent viscosity in domains with outlets containing unbounded cross sections, in the case of shear thickening viscosity. The flux assumes arbitrary given values and the growth of the cross sections are analyzed under different convergence hypotheses, inclusive the growth of Dirichlet's integral of the velocity field is deeply related the convergence hypotheses of such sections. We extend the results of the section 4 of [12, Ladyzhenskaya and Solonnikov] (for Newtonian fluids) to non-Newtonian fluids using the techniques found in [3, Dias and Santos].


*Keywords:* Power-law fluids; Ladyzhenskaya–Solonnikov problem; Non-Newtonian fluids; Shear thickening fluids

## 1. Introduction

The steady flow of the viscous incompressible fluid in many cases is modeled by the system

$$\begin{aligned} \operatorname{div}\left\{|D(\mathbf{v})|^{p-2} D(\mathbf{v})\right\} &= \delta(\mathbf{v}\cdot\nabla\mathbf{v}) + \nabla\mathcal{P} \\ \nabla\cdot\mathbf{v} &= 0\,, \end{aligned} \tag{1.1}$$

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where $\mathbf{v}$ is the velocity field and $\mathcal{P}$ is the pression function of the fluid, $D(\mathbf{v})$ is the symmetric gradient with components $D_{ij}(\mathbf{v}) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$ for $\mathbf{v} = (v_1, \dots, v_n)$, $i, j \in \{1, \dots, n\}$, $\mathbf{v} \cdot \nabla \mathbf{v} = \sum_{i=1}^{n} v_i \frac{\partial}{\partial x_i} \mathbf{v}$ is the convective term. If $p > 2$ the fluid is called *shear thickening*; and if $1 \le p < 2$, *shear thinning* (the flow is pseudo-plastic if $1 < p < 2$ and purely plastic if $p = 1$); the case $p = 2$ corresponds to the Newtonian behavior. If $\delta = 1$, (1.1) is called *Navier–Stokes power-law*; if $\delta = 0$, *Stokes power-law*. More information about the development of the equation (1.1) can be found in many papers on non-Newtonian fluids, for example Ladyzhenskaya [10,11], Marusic-Paloka [14,15], Ruzicka [18] and Frehse, Málek and Steinhauer in [6,7]. The study of this type of flow in bounded domains can be found in Lions [13], Ruzicka [18] and Frehse, Málek and Steinhauer in [6,7]. On unbounded domains there are few references, we cite Marusic-Paloka [14,15] and Dias and Santos [3].

We consider a domain $\Omega$ in $\mathbb{R}^n$, $n = 2, 3$, with a $C^\infty$ boundary, of the following type

$$\Omega = \bigcup_{i=0}^{m} \Omega_i ,$$

where $\Omega_0$ is a bounded subset of $\mathbb{R}^n$, while, in possibly different cartesian coordinate systems,

$$\Omega_i = \left\{x \equiv \left(x', x_n\right); |x'| < g_i(x_n),\ x_n > 0\right\},\quad i = 1, \dots, m ,$$

where the functions $g_i(t)$ are $C^\infty$ and satisfy the conditions

$$\begin{aligned} & g_i(t) \ge g_0 > 0 \\ & |g_i(t_1) - g_i(t_2)| \le M_i |t_1 - t_2| , \end{aligned} \tag{1.2}$$

for all $t_1, t_2, t > 0$.

We denote for $\Sigma_i(x_n)$ the cross sections of $\Omega_i$ perpendicular to the axis-$x_n$ and passing through the point $(0, x_n)$; $\Sigma$ denote a cross section any of $\Omega$, in other terms, any bounded intersection of $\Omega$ with a $(n-1)$-dimensional plane, that in $\Omega_i$ reduces to $\Sigma_i$; finally, we denote by $\mathbf{n}$ an orthonormal vector to $\Sigma$ pointing in the infinite direction of the outlets. It follows immediately from (1.2) that

$$\begin{aligned} & |g_i'(t)| \le M ,\quad \forall t > 0 \\ & g_i(t) \le 2Mt ,\quad \forall t \ge t_0 \equiv M^{-1} g_i(0). \end{aligned} \tag{1.3}$$

For $p$ greater than 2 and $i = 1, \dots, m$, by setting

$$I_i(\infty) = \int_0^\infty g_i^\lambda(t)\, dt,$$

where

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