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Attractors for impulsive non-autonomous dynamical systems and their relations

E.M. Bonotto [a,*,1], M.C. Bortolan [b], T. Caraballo [c,2], R. Collegari [a,3]

[a] *Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, Campus de São Carlos, Caixa Postal 668, São Carlos, SP, Brazil*

[b] *Departamento de Matemática, Universidade Federal de Santa Catarina, Campus Trindade, 88040-900, Florianópolis, Brazil*

[c] *Departamento de Ecuaciones Diferenciales y Análisis Numérico EDAN, Universidad de Sevilla, Sevilla, Spain*

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Abstract

In this work, we deal with several different notions of attractors that may appear in the impulsive non-autonomous framework and we explore their relationships to obtain properties regarding the different scenarios of asymptotic dynamics, such as the cocycle attractor, the uniform attractor and the global attractor for the impulsive skew-product semiflow. Lastly, we illustrate our theory by exhibiting an example of a non-classical non-autonomous parabolic equation with subcritical nonlinearity and impulses.

MSC: primary 35B41; secondary 34A37, 35R12

Keywords: Impulsive non-autonomous dynamical systems; Skew-product semiflow; Global attractor; c-Global attractor; Uniform attractor; Cocycle attractor

* Corresponding author.

E-mail addresses: ebonotto@icmc.usp.br (E.M. Bonotto), m.bortolan@ufsc.br (M.C. Bortolan), caraball@us.es (T. Caraballo), collegari@gmail.com (R. Collegari).

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1. Introduction

What are the differences that appear when we change from an autonomous equation to an non-autonomous one? Does the asymptotic behavior of the solutions become different?

This change may be very underrated and our first answer may be negative. We might believe that there are not many changes in the behavior of the solutions of autonomous and non-autonomous equations. As one may see in [10,11], this is not the case. In fact, there are infinitely many differences between these two cases. To illustrate this difference, let us consider a general differential equation of the form

$$\begin{cases} \dot{u} = f(t,u), \ t > s, \\ u(s) = u_0 \in X, \end{cases} \tag{1.1}$$

where X is a Banach space and $f : \mathbb{R} \times D \to X$ is a map, where D is an open subset of X, for which there exists a unique solution $[s, +\infty) \ni t \mapsto u(t, s, f, u_0) \in X$ of (1.1) defined for all $t \geqslant s$, for each $u_0 \in X$ and $s \in \mathbb{R}$.

Thanks to the uniqueness of solution, one can see that when f is time independent, that is, $f(t,x) = f(x)$ for all $t \in \mathbb{R}$, we have $u(t, s, f, u_0) = u(t-s, 0, f, u_0)$ and the asymptotic behavior of solutions can be studied when $t \to +\infty$ (that is, considering the evolution of the solution as the final time evolves) or making $s \to -\infty$ (which is equivalent to consider the behavior of the solution as we take earlier and earlier initial times). In this case, these two scenarios coincide and give us the same description.

However, if f is time dependent then these two situations give rise to completely different behaviors. We may study the asymptotic behavior with respect to the elapsed time $t-s$ or with respect to s (when $s \to -\infty$ and t is arbitrary but fixed). These are called, respectively, forward and pullback dynamics and are, in general, unrelated. It is natural that they are unrelated, for instance, the set of vector fields driving the solution may be completely different. We have one vector field $f(t, \cdot)$ for each time $t \in \mathbb{R}$.

There is no reason for this to be different in the impulsive case. We know now, after the previous discussion in [4], that the behavior of impulsive solutions in the non-autonomous case is much richer (and harder to analyze) than in the autonomous case. Hence, bearing this in mind, we may wonder about the relationships among the several different scenarios that appear in the non-autonomous impulsive case.

Note that the theory described in [10,11,14,15] has, so far, no analogous when it comes to the impulsive framework. Therefore, this paper shall be devoted to relate the several different kinds of attractors that come to play when dealing with non-autonomous impulsive dynamical systems.

Moreover, the results presented in this paper are totally different from the results which deal with random dynamical systems, where the impulses occur in time. Indeed, the results of this paper concern with impulses at variable times that depend on the phase space (impulses "occur" in space). Impulses that vary in time are more attractive due to their complexity and applicability in real world problems, see for instance [5–7]. As an example, we may cite the billiard-type system which can be modeled by differential systems with impulses acting on the first derivatives of the solutions. Indeed, the positions of the colliding balls do not change at the moments of impact (impulse), but their velocities gain finite increments (the velocity will change according to the position of the ball). The reader may consult [28] for the study of pullback attractors of non-autonomous random dynamical systems.

In the following we describe the organization of the paper and the main results.

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